22, 23 y 24 de Octubre de 2008. El Escorial (Madrid)

Cuarta reunión de la red temática Dance

Ddays 2008

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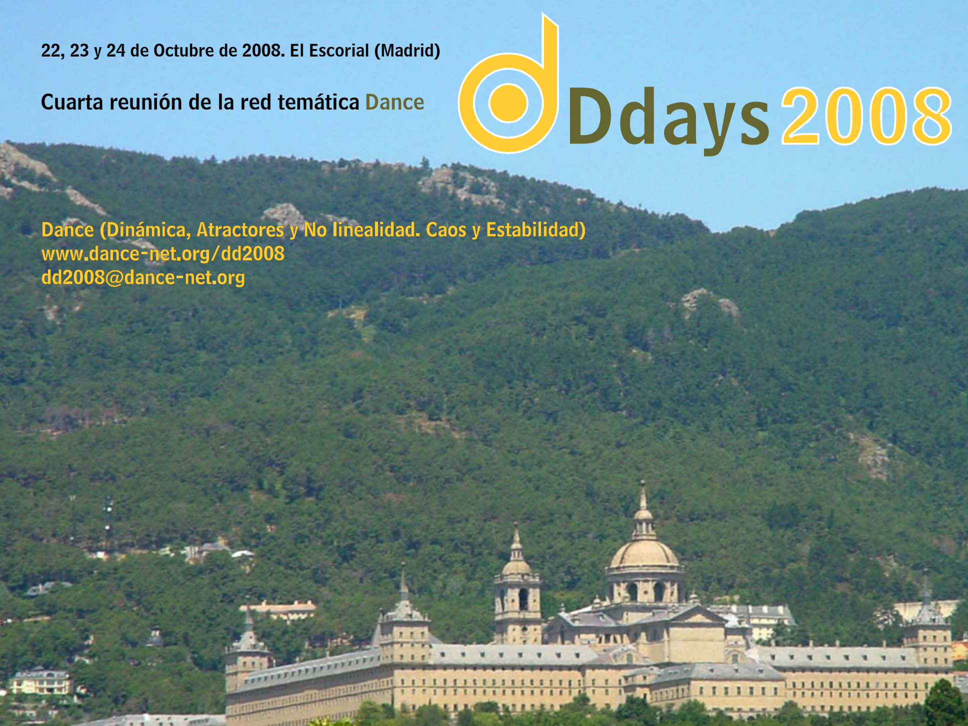

Analytical and Numerical Tools for the Study of Normally Hyperbolic Invariant Manifolds in Hamiltonian Systems and their Associated Dynamics

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DDAYS 2008

Outline

Spatial RTBP

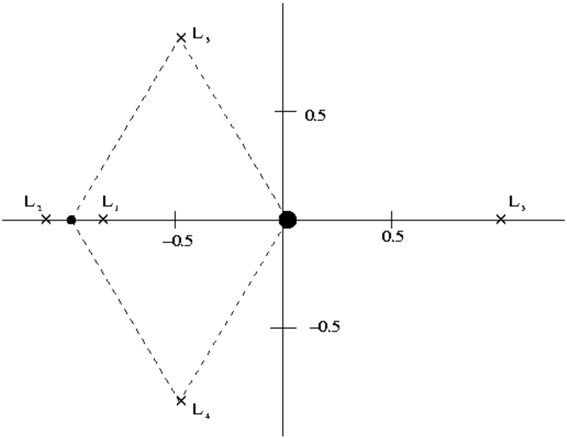


Figure: The five equilibrium points of the RTBP.

Spatial RTBP

- L_1 and L_2 are center $\times$ center $\times$ saddle.
- $\exists$ Normally hyperbolic center manifold Λ (locally).
- $\exists$ Stable/unstable invariant manifolds $W^s(\Lambda)$, $W^u(\Lambda)$.
- Question: Arnold's diffusion? Symbolic dynamics? Mission design?

Parallel Computation of Normal Forms

- Normal form of Hamiltonian system around equilibrium point.
- Efficiency of normal form methods: time and memory.
- 2 assymptotic scenarios:
  - ODE setting (hard limit: time).
  - PDE setting (hard limit: memory).
- Parallel algorithm.
- Implementation (based on À. Jorba).

Parallel Computation of Normal Forms: Results

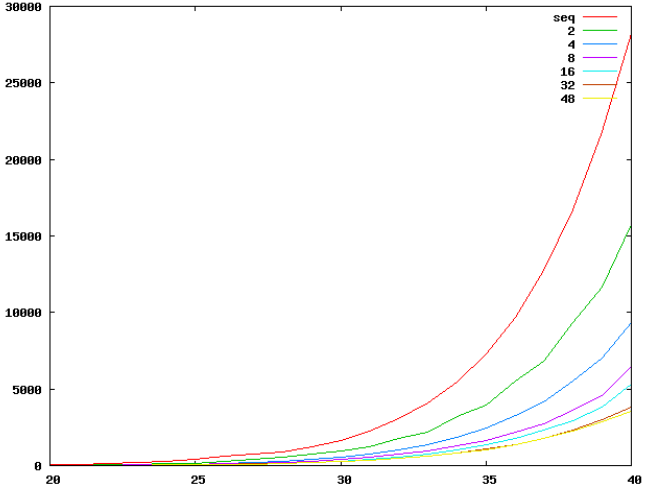
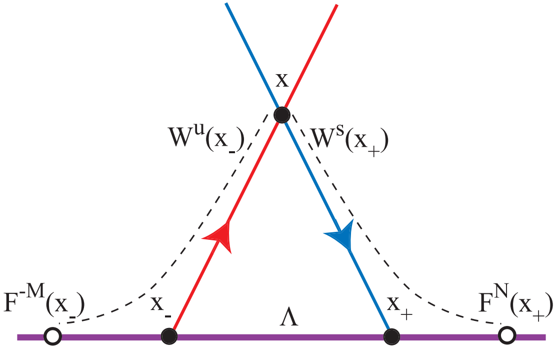


Figure: Computation time (in seconds) vs. normal form degree.

Scattering Map: Definition [DLIS]

- $S\colon \Lambda \to \Lambda, \qquad S(x_-) = x_+$.

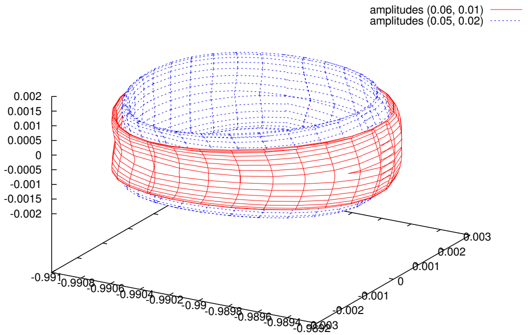


Local Approximation of Dynamics

- High-order truncated normal form around L:

$$H = H_N(q_1 p_1, q_2, p_2, q_3, p_3) + R_{N+1}.$$

- Normally hyperbolic center manifold Λ.



Local Approximation of Dynamics

- High-order truncated normal form around L:

$$H = H_N(q_1 p_1, q_2, p_2, q_3, p_3) + R_{N+1}.$$

- Local st/unst invariant manifolds

$$W^s_{\mathrm{loc}}(\Lambda), W^u_{\mathrm{loc}}(\Lambda).$$

- Local st/unst preserved fibres

$$W^s_{\mathrm{loc}}(x_+), W^u_{\mathrm{loc}}(x_-).$$

Numerical Globalization

- Numerical integration of RTBP (RK-PD order 8-9).
- Global st/unst invariant manifolds

$$W^s(\Lambda), W^u(\Lambda).$$

- Preservation of fibres:

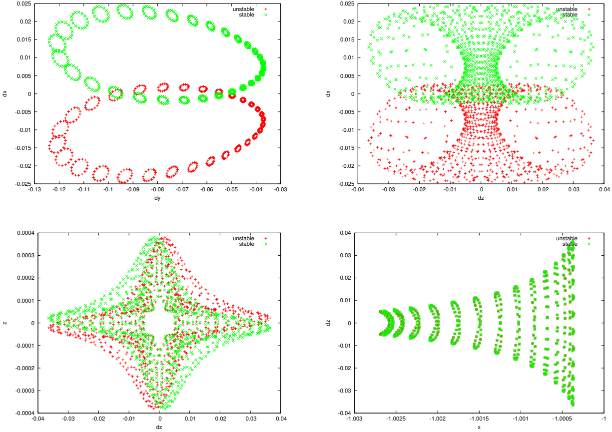
$$x \in W^s(x_+) \Rightarrow \varphi^t(x) \in W^s(\varphi^t(x_+)).$$

- Global st/unst preserved fibres

$$W^s(x_+), W^u(x_-).$$

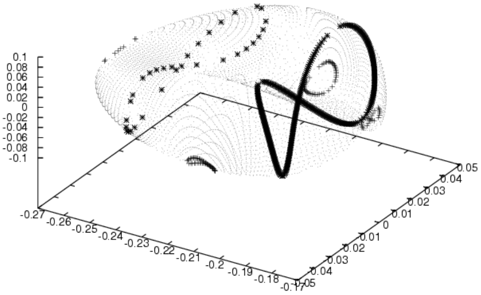
Homoclinic Intersection $W^s(\Lambda) \cap W^u(\Lambda)$

- Globalize st/unst manifolds up to a surface of section.



Homoclinic Intersection $W^s(\Lambda) \cap W^u(\Lambda)$

- Numerical intersection using Newton's method (J. Masdemont).



- Parallel computation.

Reduced Scattering Map

- Homoclinic intersection + fibers $\Rightarrow$ Scattering map

$$S\colon \Lambda \to \Lambda \qquad \text{3D}.$$

- 2D Poincaré section $\Sigma \subset \Lambda$

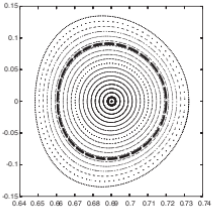


Figure: Poincaré section $\Sigma = \{(\varphi_1, \varphi_2, \alpha) \in \Lambda \mid \varphi_1 = 0\}$

- Reduced scattering map

$$S\colon \Sigma \to \Sigma \qquad \text{2D}.$$

Scattering Map: Results

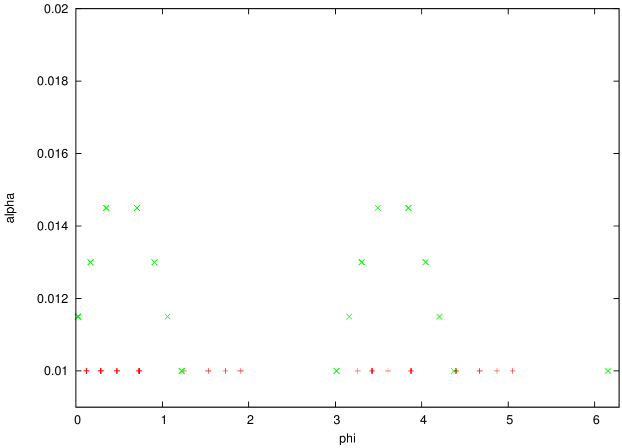


Figure: An invariant torus and its image under the scattering map.

Scattering Map: Results

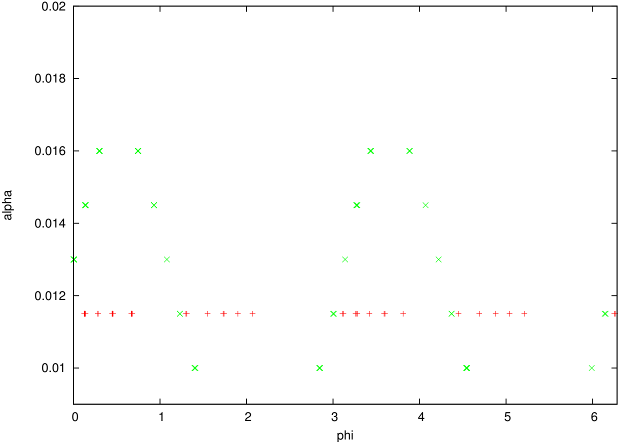
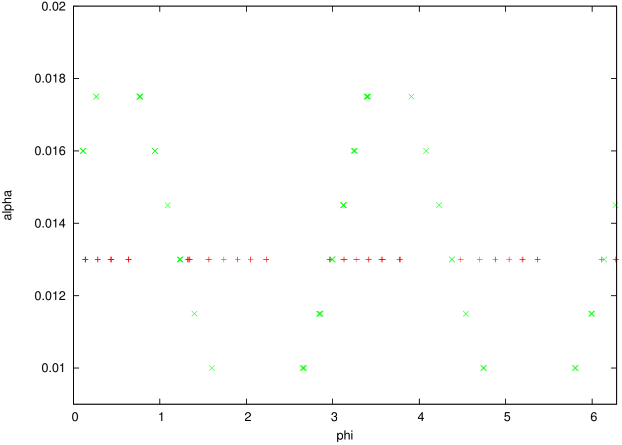


Figure: An invariant torus and its image under the scattering map.

Scattering Map: Results



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Scattering Map: Results

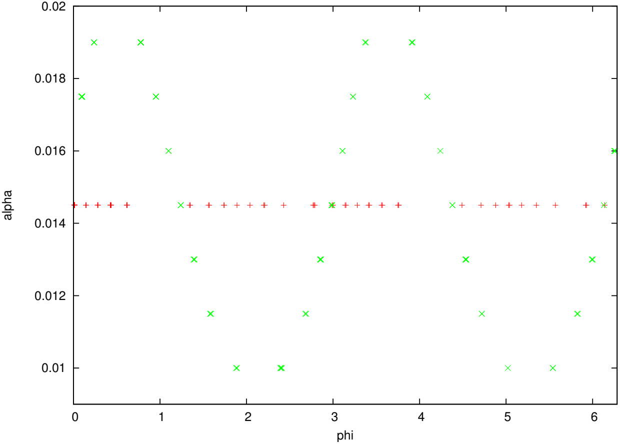


Figure: An invariant torus and its image under the scattering map.

Arnold's Diffusion (with M. Gidea)

- Low-dimensional model:
  - Inner map
    $$T: \Sigma \to \Sigma.$$
  - Outer map
    $$S: \Sigma \to \Sigma.$$

Topological Shadowing

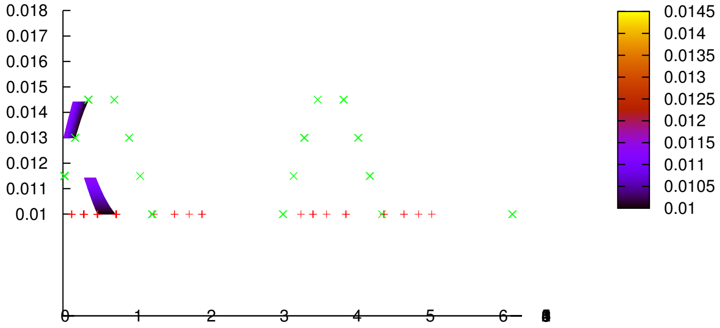
Lemma

Let $\{R_i\}$ be a sequence of 2D windows on Σ.
Assume the following:

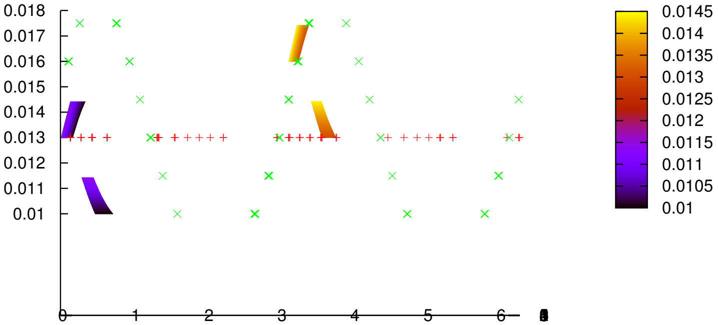
1. $\forall i$, *window R_{2i} is correctly aligned with window R_{2i+1} under the outer map S.*
2. $\forall i$, *window R_{2i+1} is correctly aligned with window R_{2i+2} under some iterate of the inner map T.*

Then, $\exists$ a true orbit passing near all the windows.

Correctly Aligned Windows



Correctly Aligned Windows



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