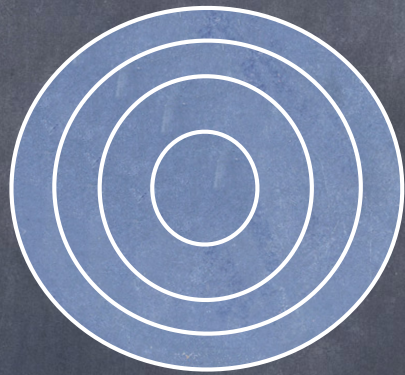
Angle - action



- Courbes fermées
- Transformation canonique particulière

$$(q,p) \rightarrow (\psi, J)$$

$$\mathcal{K} = K(\psi, J) = K(-, J)$$

- L'action = aire sous-tendue par l'orbite

$$J = \mathrm{cste} = \frac{1}{2\pi}\int p\,dq$$

- L'angle = fonction linéaire du temps

$$\psi = \frac{dK}{dJ}\,t + \psi_0$$

Angle action du pendule

H = $p^2/2 - \delta \cos q$

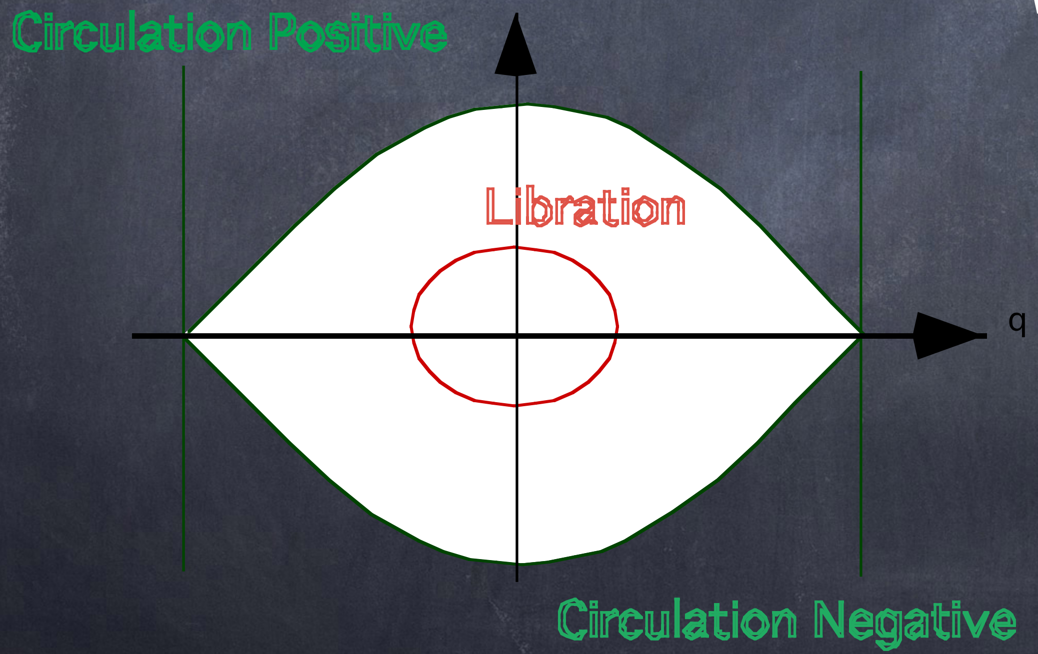
$$J = \frac{1}{2\pi}\oint p\,dq$$

$$= \frac{8\sqrt{\delta}}{\pi}\left((k^2-1)\,\mathcal{K}(k) + \mathcal{E}(k)\right) \quad \text{en libration}$$

$$\text{avec} \quad k = \sqrt{\frac{1}{2}+\frac{H_0}{2\delta}}$$

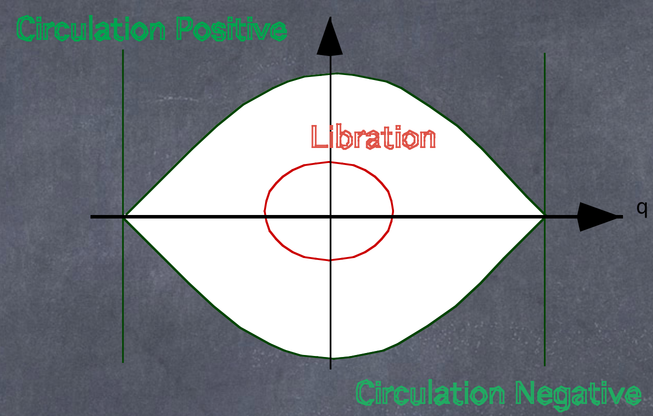
$$= \frac{2}{\pi}\sqrt{2H_0+2\delta}\;\mathcal{E}(k)) \quad \text{en circulation}$$

$$\text{avec} \quad k = \sqrt{\frac{2\delta}{H_0+2\delta}}$$



Autour d'un équilibre stable

Angle action de l'oscillateur harmonique



$$
\begin{aligned}
H_{Loc} &= H_{eq} + \frac{\partial H}{\partial q}(q-q_{eq})\frac{\partial H}{\partial p}(p-p_{eq}) \\
&+ \frac{1}{2}\,\Big(\frac{\partial^2 H}{\partial q^2}(q-q_{eq})^2 + \frac{\partial^2 H}{\partial p^2}(p-p_{eq})^2 + 2\,\frac{\partial^2 H}{\partial p \partial q}(q-q_{eq})(p-p_{eq}) \\
&+ \ldots \\
&\equiv a\,\Delta_q^2 + 2\,b\,\Delta_q\,\Delta_p + c\,\Delta_p^2 + \ldots
\end{aligned}
$$

$$
\begin{aligned}
\xi &= \Delta_q \cos\theta + \Delta_p \sin\theta \\
\eta &= -\Delta_q \sin\theta + \Delta_p \cos\theta
\end{aligned}
$$

$$H_{loc} = A\,\xi^2 + B\,\eta^2 + \ldots$$

$$
\begin{aligned}
\xi &= \alpha\,\sqrt{2J}\,\cos\psi \\
\eta &= \frac{1}{\alpha}\,\sqrt{2J}\,\sin\psi
\end{aligned}
$$

$$H_{loc} = w\,J + \ldots$$

Fréquence à l'équilibre

$$\dot{\psi} = w = \frac{\partial H_{loc}}{\partial J}$$

Modèles de résonances

$$\mathcal{K} = \beta\, S^2 + \alpha\, S + \epsilon\ \cos\sigma$$ Pendule

$$\mathcal{K} = \beta\, S^2 + \alpha\, S + \epsilon\, \sqrt{2S}\cos\sigma$$ SFMR : i=1

$$\mathcal{K} = \beta\, S^2 + \alpha\, S + \epsilon\, 2S\cos 2\sigma$$ i=2

$$\mathcal{K} = \beta\, S^2 + \alpha\, S + \epsilon\, (\sqrt{2S})^3\cos 3\sigma$$ i=3

$$\mathcal{K} = \beta\, S^2 + \alpha\, S + \epsilon\, \sqrt{2S}\cos\sigma + \eta\, 2S\cos 2\sigma$$

SFMRAS : i=1

$$\begin{aligned}
\alpha &= \alpha(N) = \alpha(a,e)\\
\beta &= \beta(N) = \beta(a,e)\\
\epsilon &= \epsilon(N) = \epsilon(a,e)\\
\eta &= \eta(N) = \eta(a,e)
\end{aligned}$$

Suitable change of scales + phases

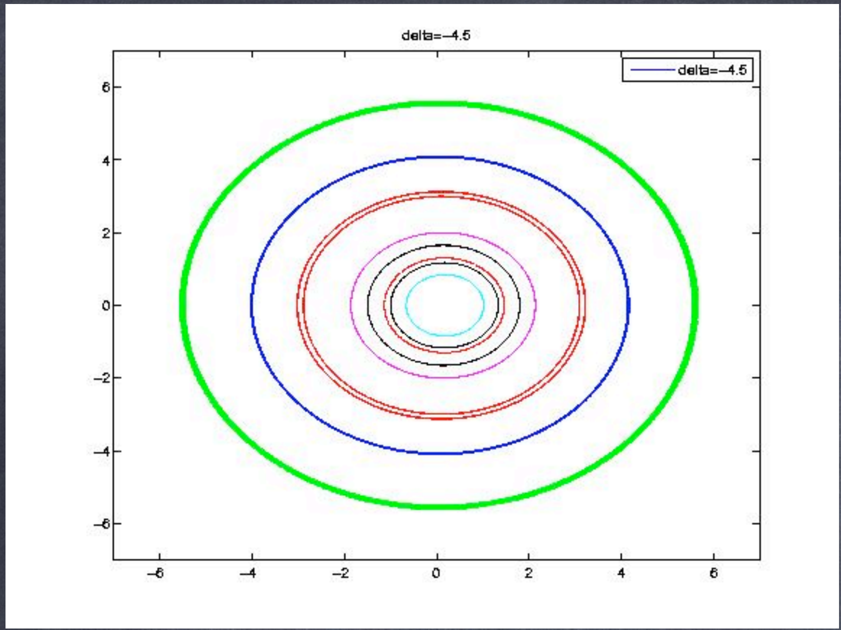
$$\begin{aligned} R &= a\,S \\ r &= b\,\sigma + c \\ \tau &= d\,t \\ K &= s\,H \end{aligned}$$

$$K = -3\,(\delta+1)\,R + R^2 - 2\,\sqrt{2R}\,\cos r$$

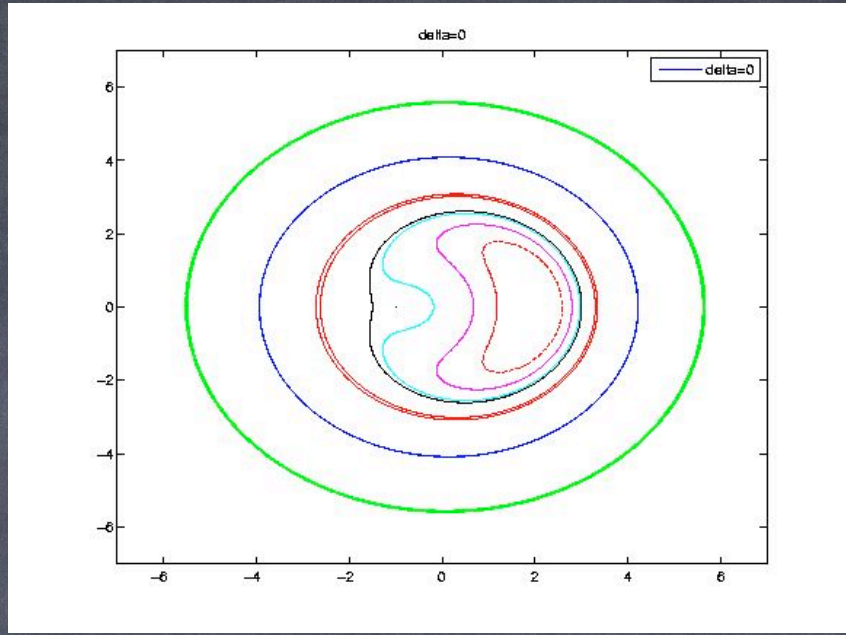
$$\begin{aligned} x &= \sqrt{2\,R}\,\cos r \\ y &= \sqrt{2\,R}\,\sin r \end{aligned}$$

$$K = -3\,(\delta{+}1)\,\frac{x^2+y^2}{2} + \left(\frac{x^2+y^2}{2}\right)^2 - 2\,x$$

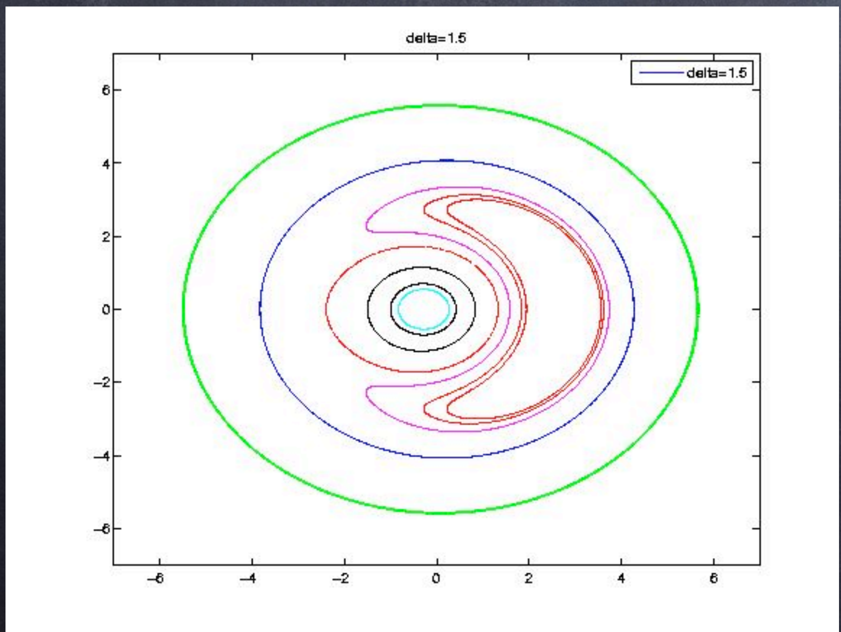
Equilibria : y = 0 and a cubic equation in x

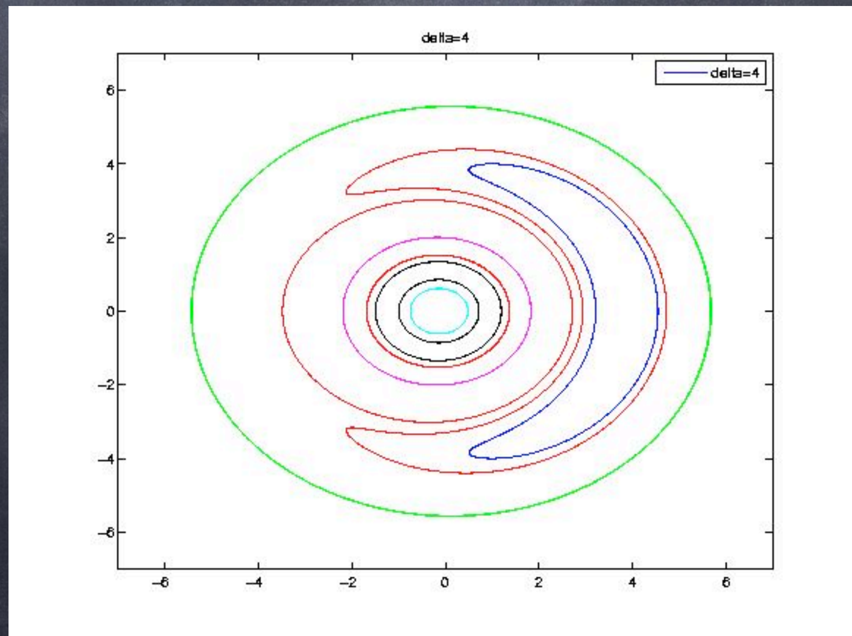


Variations périodiques
de l'excentricité



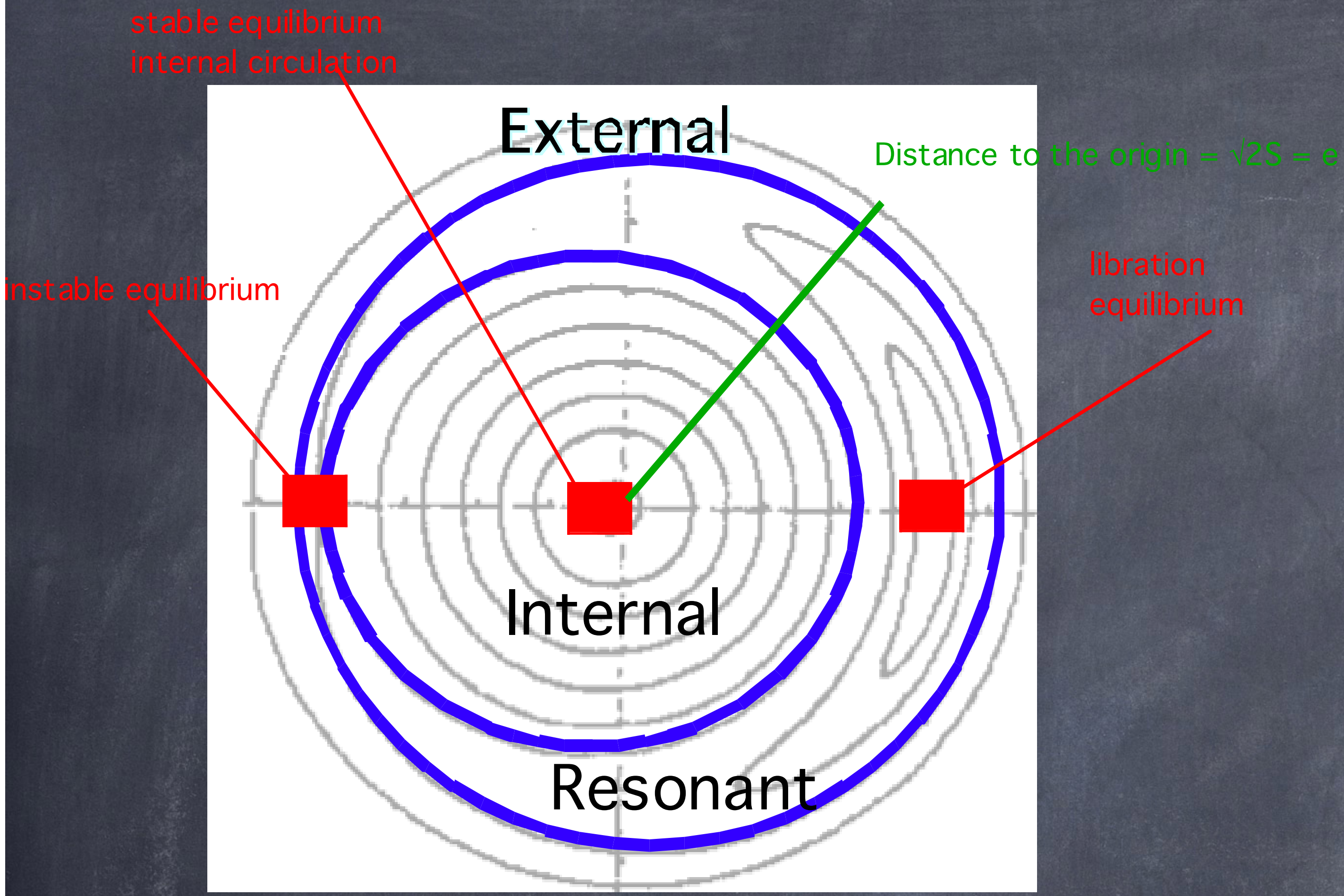
Larges excursions
en excentricité





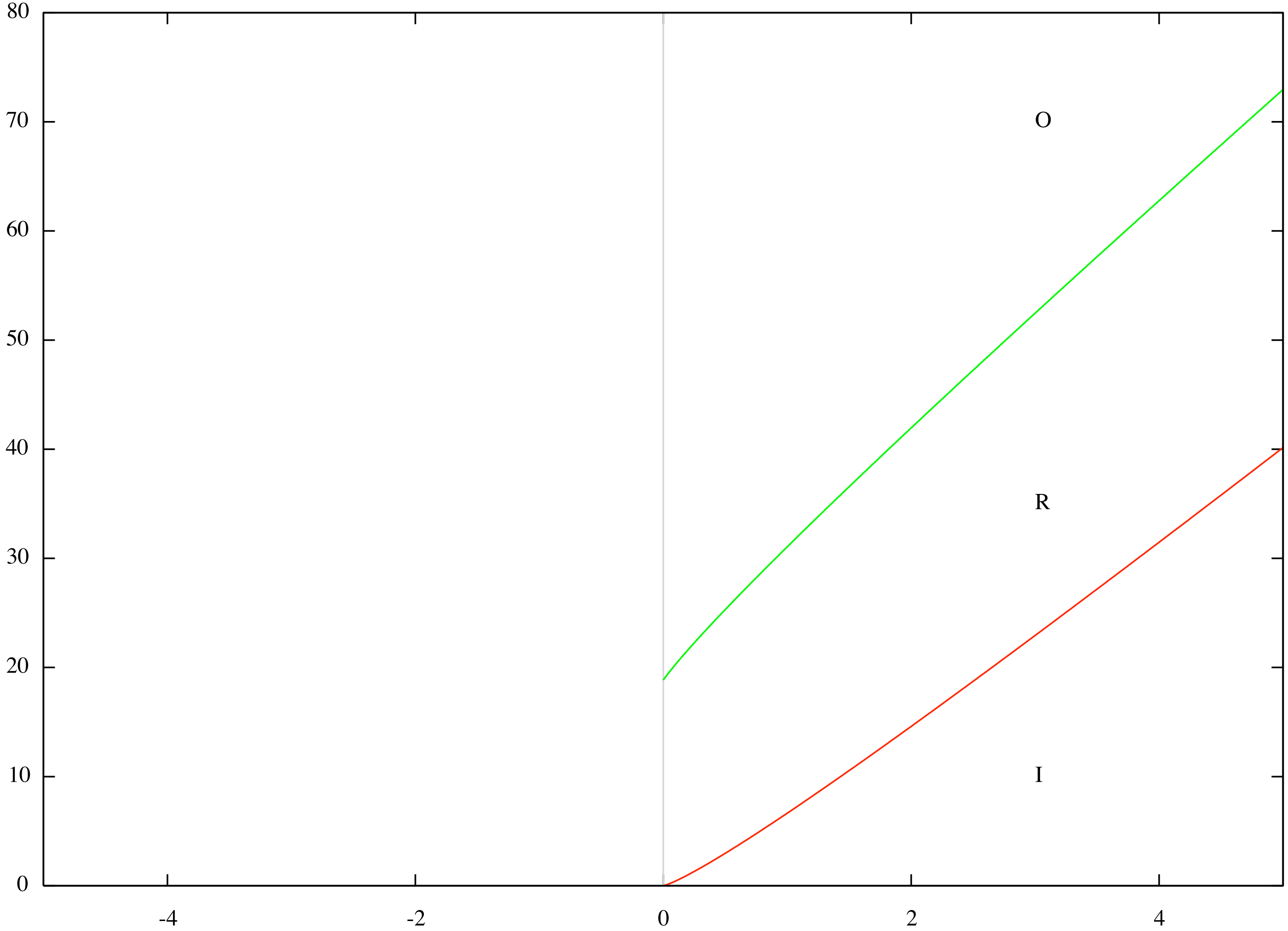
Equilibria known as functions of the parameter
The hamiltonian level of the critical curve is also known
The positions of the two limits also
The a-e V shape is based on this calculateur

Area enclosed by each trajectory is implicit or numerical
The two critical areas can be calculated
Creation of a new reapersentation of the orbits : parameter - area (index)



$$
\begin{aligned}
\text{zone interne} \quad &: \quad 0 \leq A \leq A_1(\delta) \\
\text{zone résonante} \quad &: \quad 0 \leq A \leq A_2(\delta) - A_1(\delta) \\
\text{zone externe} \quad &: \quad A \geq A_2(\delta)
\end{aligned}
$$

$$\begin{aligned} \text{zone interne} &\quad : \quad 0 \leq \theta = A \leq A_1(\delta) \\ \text{zone résonante} &\quad : \quad A_1(\delta) \leq \theta = A + A_1(\delta) \leq A_2(\delta) \\ \text{zone externe} &\quad : \quad \theta = A \geq A_2(\delta) \end{aligned}$$



$$K = 2\,R^2 - (2\,\delta + 1)R + R\cos 2r$$
$$= (\frac{x^2+y^2}{2})^2 - (2\,\delta + 1)\,\frac{x^2+y^2}{2} + \frac{x^2-y^2}{2}$$

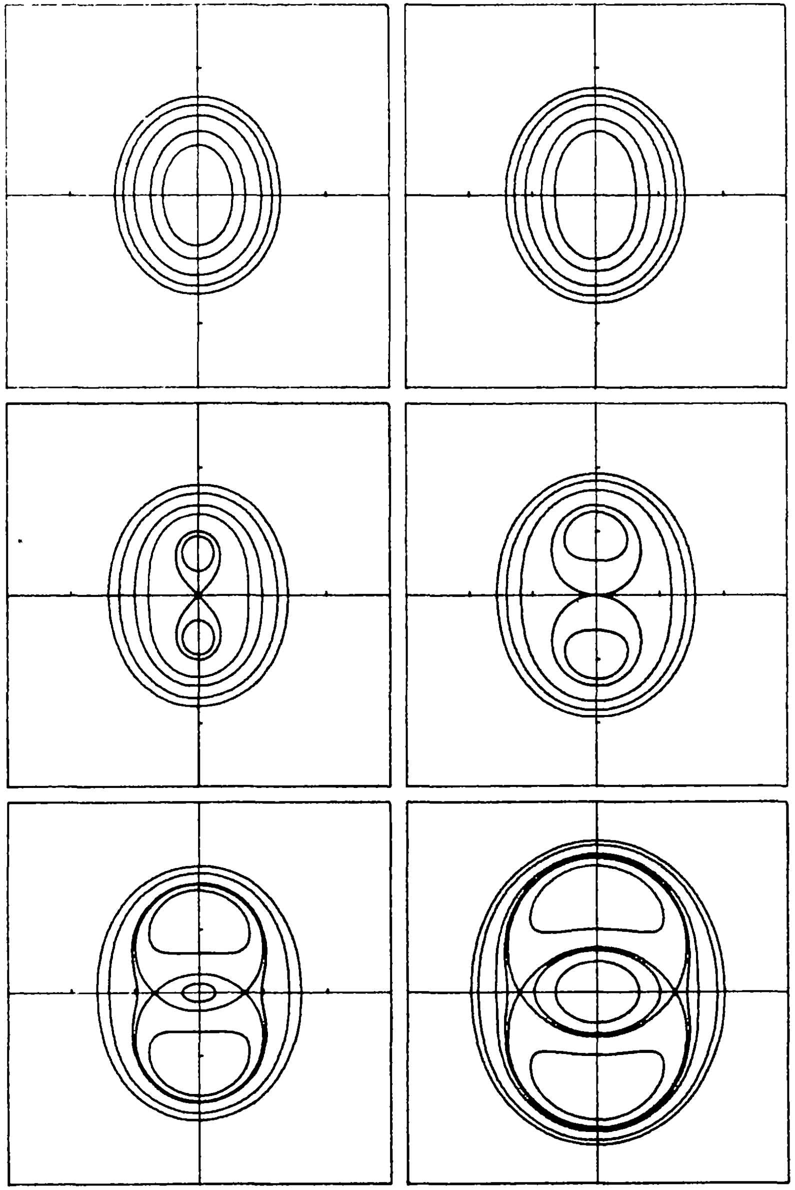


Fig. 1. The phase space for the (j + 2)/j case.

Equilibria : symmetry in x and y

Resonances of order 2

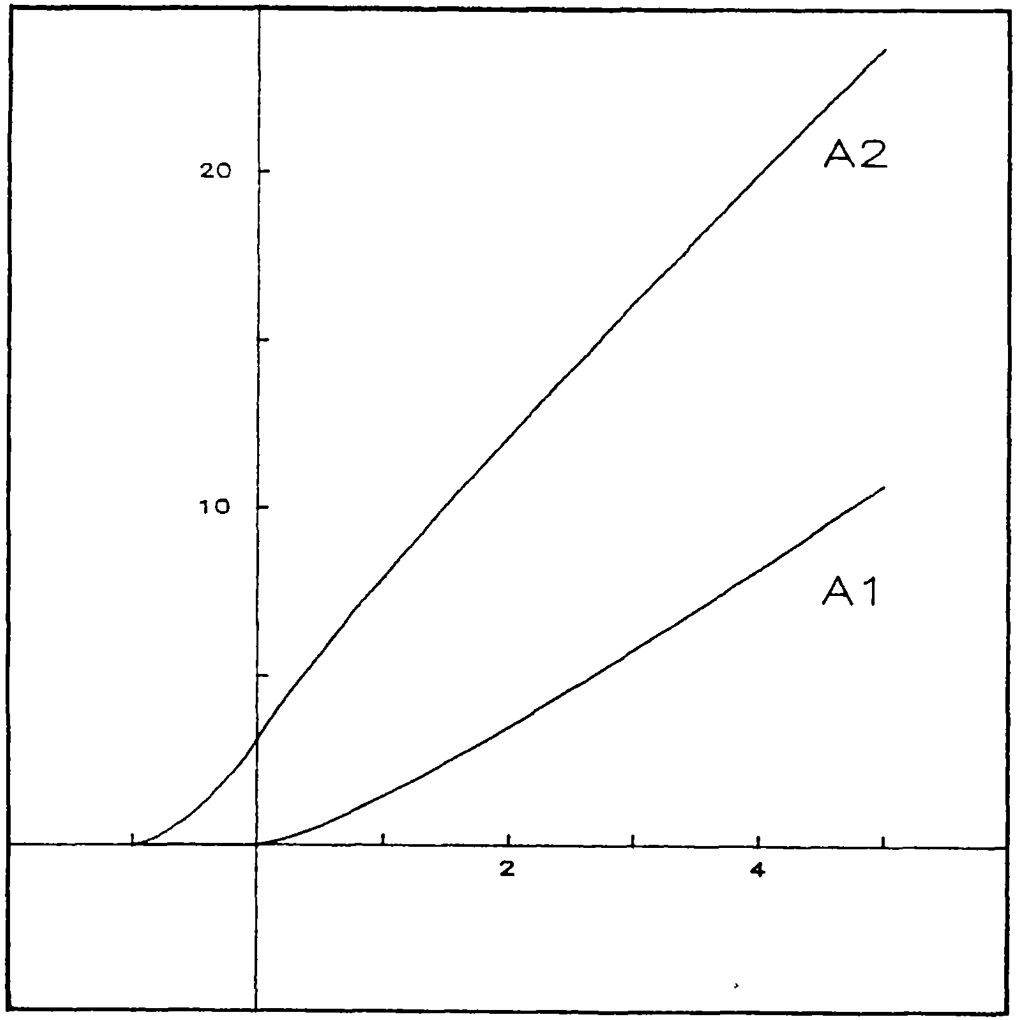


Fig. 3. The area index diagram for the (j + 2)/j case.

Resonances of order 3

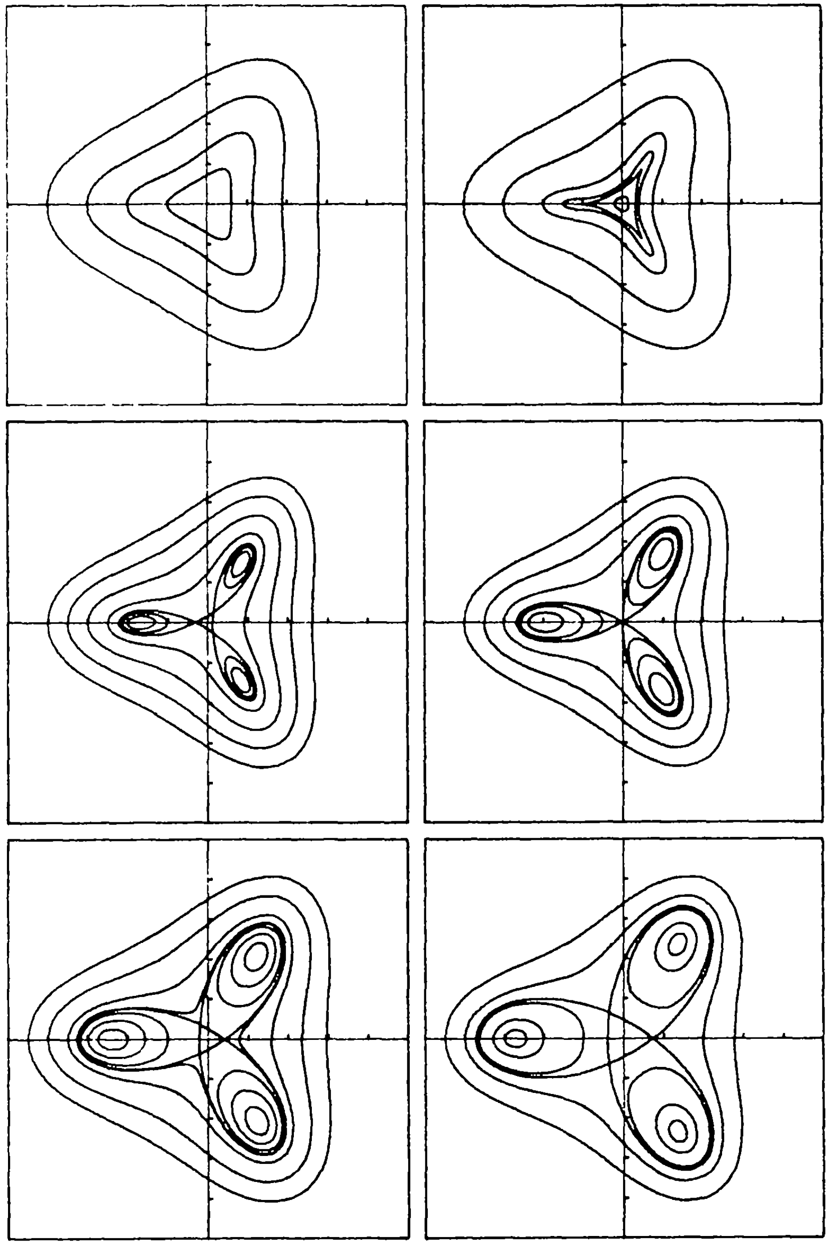


Fig. 4. The phase space for the (j + 3)/j case.

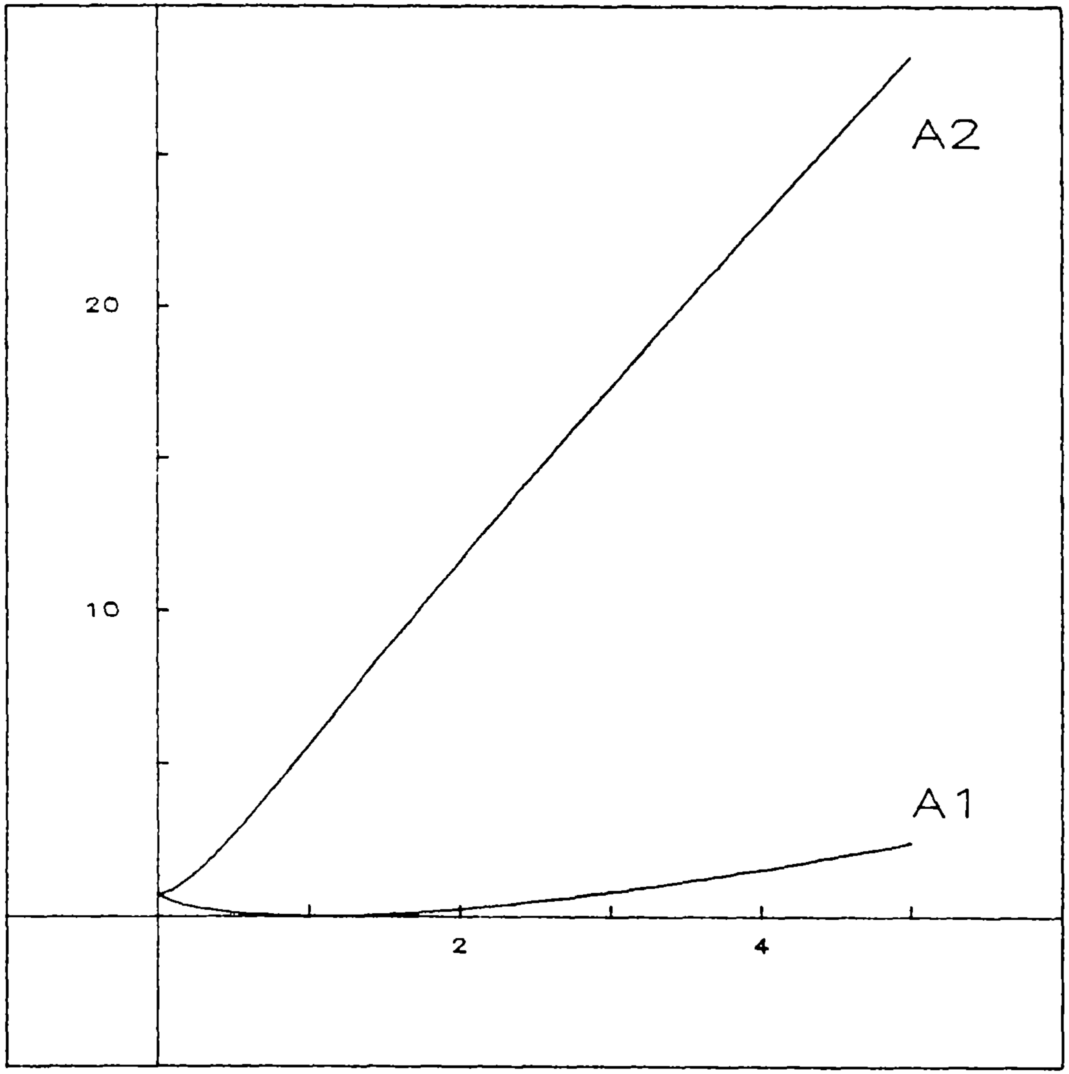
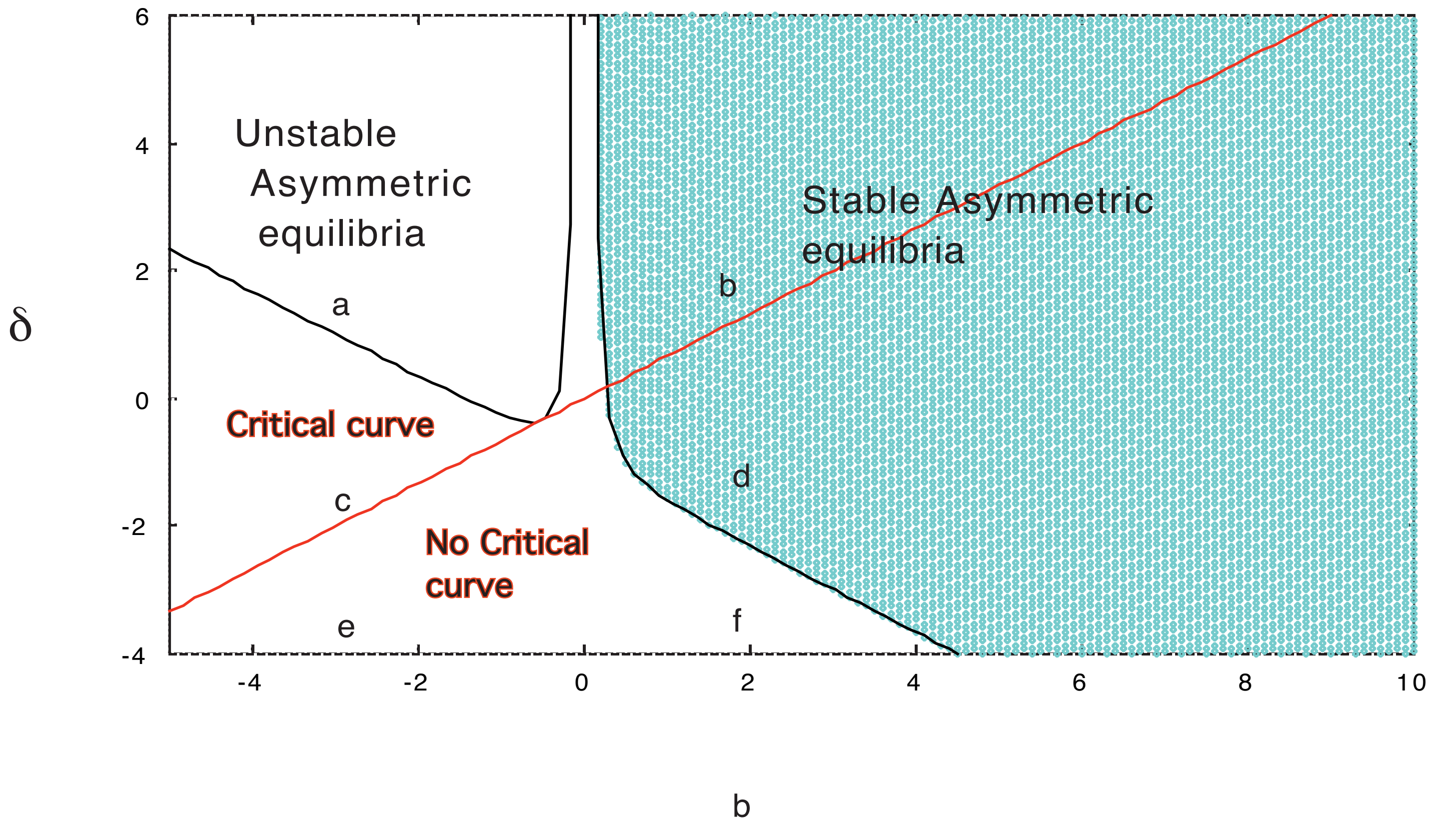
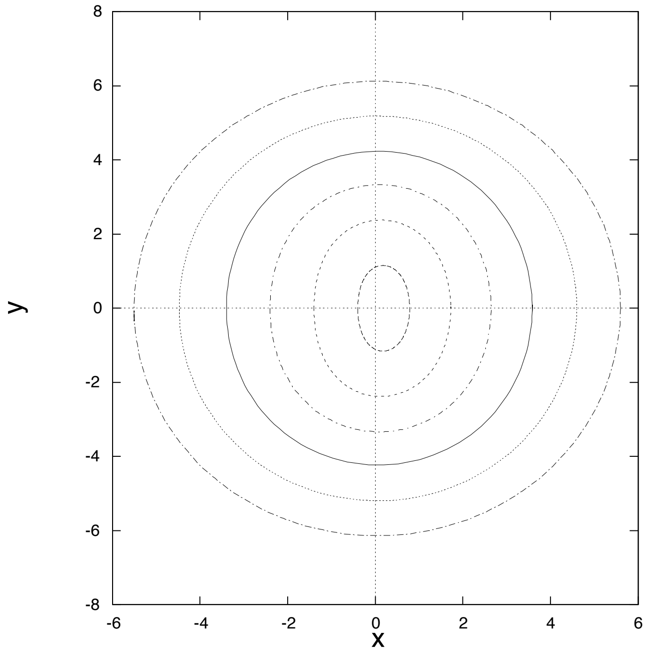
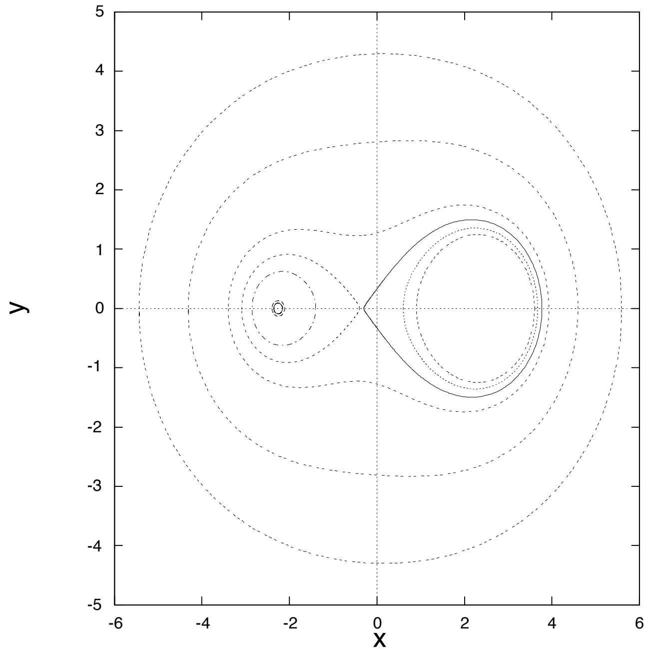


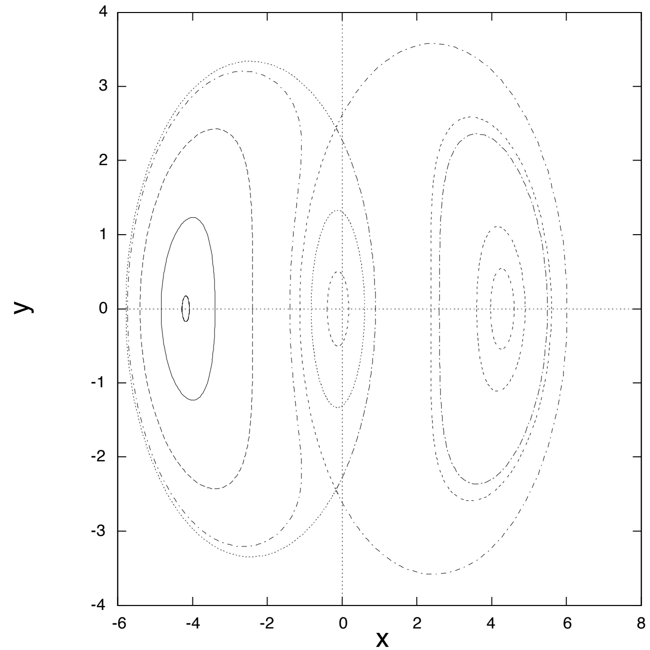
Fig. 6. The area index diagram for the (j + 3)/j case.



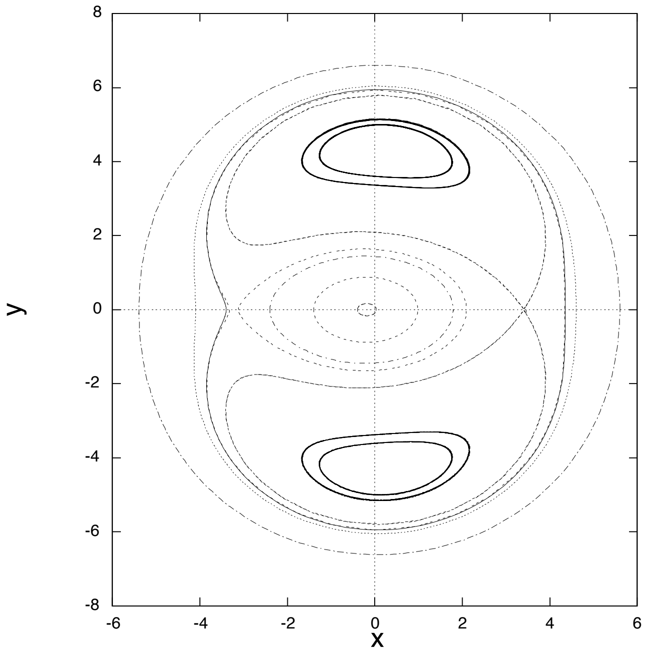
$$K = -3\,(\delta + 1)\,R + R^2 - 2\sqrt{2R}\cos r + b\;2R\cos 2R$$

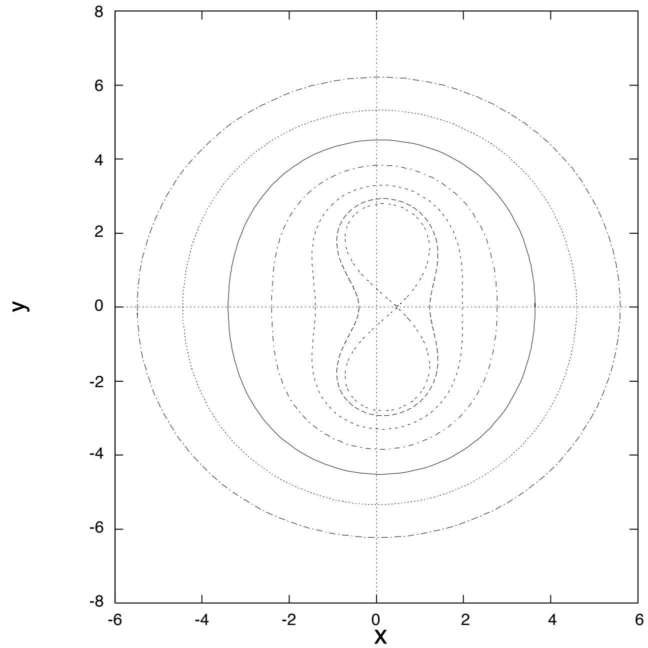




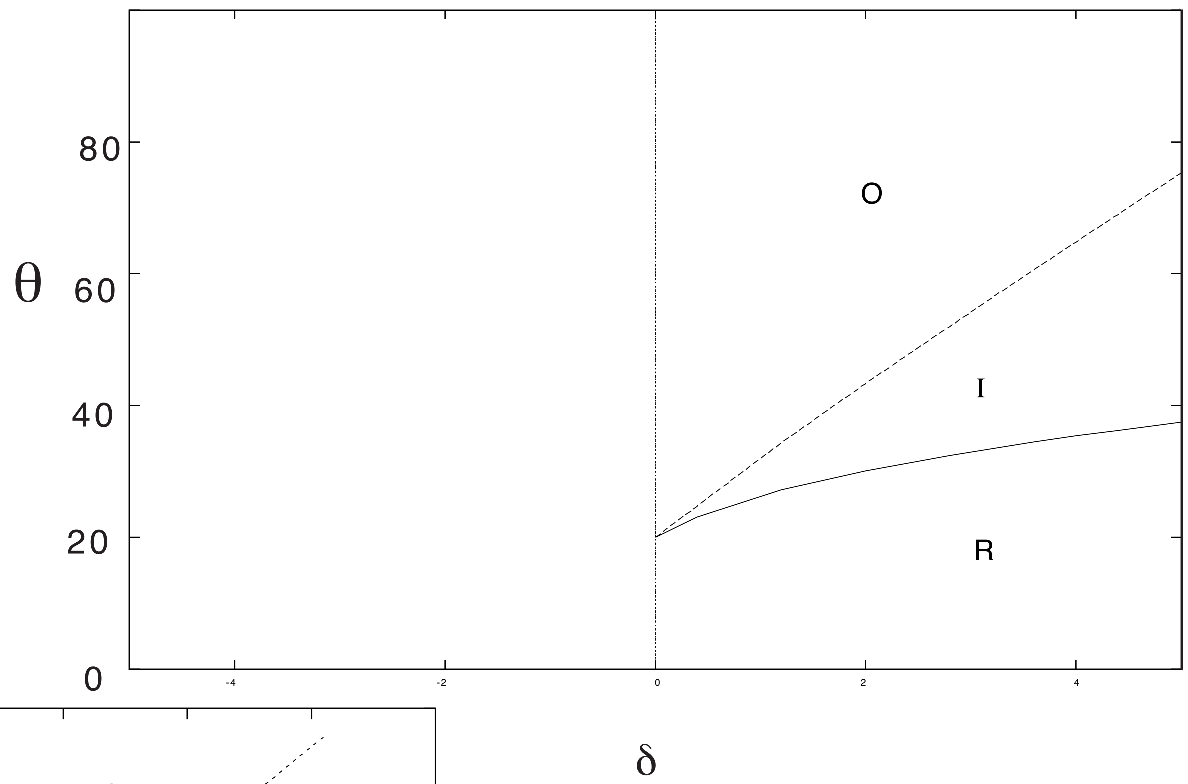


$$K \quad = \quad -3\,(\delta+1)R + R^2 - 2\,\sqrt{2R}\,\cos 2r + b\;2R\;\cos 2r$$

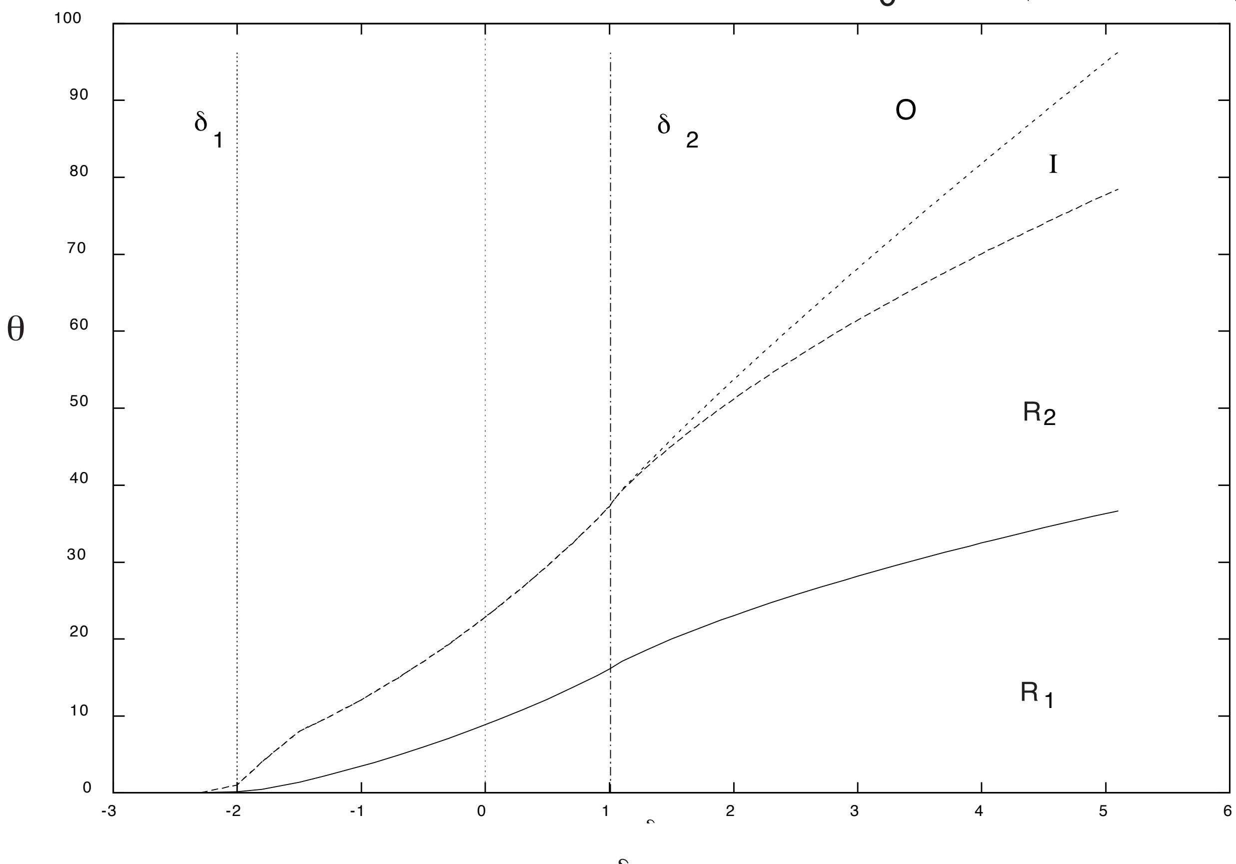




SFMRAS
External 2:1


θ
80
60
40
20
0
-4
-2
0
2
4
O
I
R
δ

b=0


θ
100
90
80
70
60
50
40
30
20
10
0
-3
-2
-1
0
1
2
3
4
5
6
δ 1
δ 2
O
I
R2
R1

b=-3

Systèmes hamiltoniens

- Hamiltonien à 1 degré de liberté
- Equations différentielles associées
- Transformation canonique
- Conservation de la dynamique symplectique

Variables Moments

$$\mathcal{H} = H(q,p)$$

$$\begin{aligned} \frac{dq}{dt} &= \frac{\partial H}{\partial p} \\ \frac{dp}{dt} &= -\frac{\partial H}{\partial q} \end{aligned}$$

$$(q,p) \longrightarrow (Q,P)$$

$$\begin{aligned} \mathcal{K} &= K(P,Q) \\ &= H(q(Q,P),p(Q,P)) \end{aligned}$$

$$\begin{aligned} \frac{dQ}{dt} &= \frac{\partial K}{\partial P} \\ \frac{dP}{dt} &= -\frac{\partial K}{\partial Q} \end{aligned}$$

Introduction du temps

- Hamiltonien dépendant du temps

$$\mathcal{H} = H(q, p, t)$$

- Transformation canonique dépendant du temps

$$(q, p) \longrightarrow (Q, P)$$

- Fonction génératrice S

$$\mathcal{S} = S(q, P, t)$$

$$\begin{aligned} p &= \frac{\partial S}{\partial q} \\ Q &= \frac{\partial S}{\partial P} \end{aligned}$$

- Nouvel hamiltonien

$$\begin{aligned} \mathcal{K} &= K(Q, P, t) \\ &= H(q(Q, P, t), p(Q, P, t)) + \frac{\partial S}{\partial t} \end{aligned}$$

Adiabatic Invariant

$$\begin{aligned} H &= H_0(q,p) + H_1(q,p) \\ &= H_0(J) + H_1(J,\psi) \end{aligned}$$

Action - angle

$$\Downarrow \quad \text{moyenne sur } \psi$$

$$\bar{H} = \bar{H}_0(\bar{J}) + \bar{H}_1(\bar{J}) + \ldots + \bar{H}_n(\bar{J}) + \bar{H}_{n+1}(\bar{J},\bar{\psi})$$

$$\begin{aligned} \dot{\bar{J}} &= -\frac{\partial \bar{H}}{\partial \bar{\psi}} \equiv 0 \\ \dot{\bar{\psi}} &= \frac{\partial \bar{H}}{\partial \bar{J}} = \omega(\bar{J}) \end{aligned}$$

$$\begin{aligned} J &= \bar{J} + \mathcal{A}_1(\bar{J},\bar{\psi}) \\ \psi &= \bar{\psi} + \mathcal{B}_1(\bar{J},\bar{\psi}) \end{aligned}$$

order n

order 1

Classical Adiabatic Invariant Theory

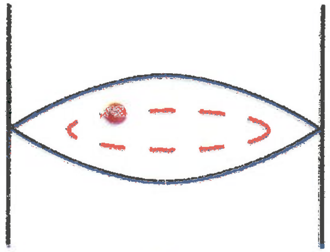
$H(q, p, \lambda)$

with $\lambda = \varepsilon t$

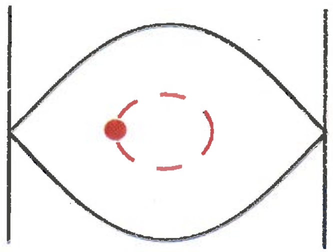
pendulum for instance

$= \frac{1}{2}p^2 - b(\lambda)\cos q$

$t_1 - t_0 = O(1/\varepsilon)$



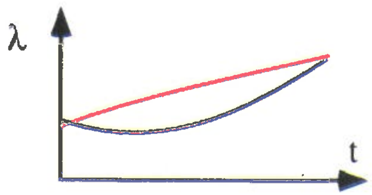
$b = b_0$



$b = b_1$

Guiding trajectory

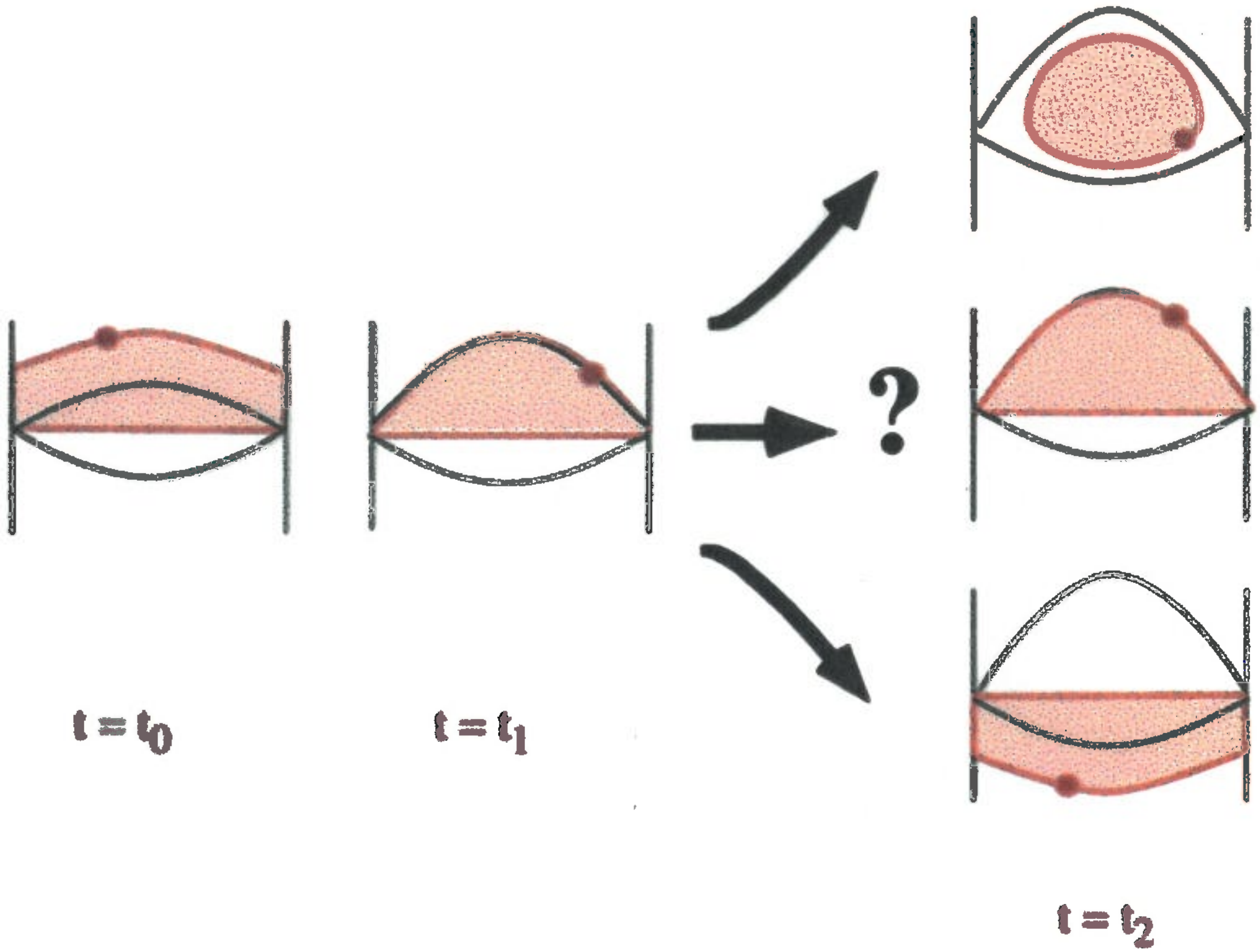
- The particle always stay close (within ε) of a trajectory of the "frozen system"
- The information about phase is lost.
- The path $\lambda(t)$ doest not matter as long as the end points are the same and the process is "slow":



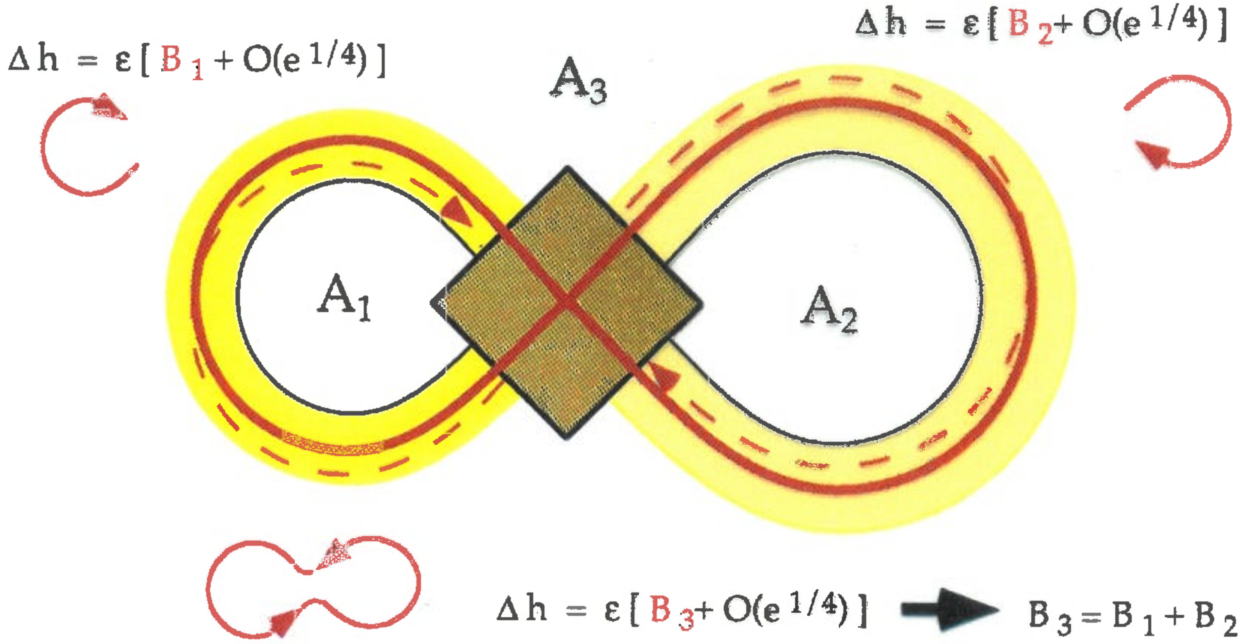
$d^n \lambda / d\varepsilon^n \sim \varepsilon^n$ for $n < N$

Lun. 1

Separatrix crossing



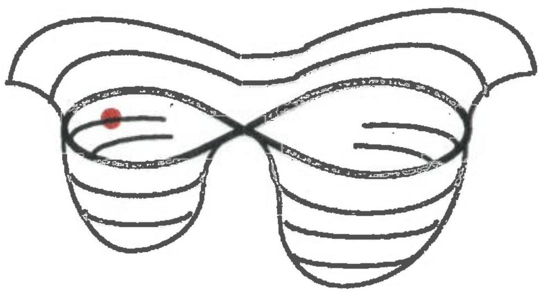
Neoadiabatic theory



$$B_i = \text{sign}(h_i) \frac{\partial \, \text{area}(A_i)}{\partial \lambda}$$

Examine the various cases . . .

assume $h_1 < 0, h_2 < 0, h_3 > 0$, and $B_1 > 0$

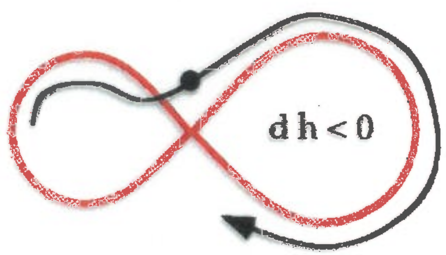


$B_1 > 0$, $B_2 < 0, B_3 < 0$

after • one looses energy
and go down the well

A_1 and A_3 shrink A_2 grows.

$B_1 > 0$, $B_2 > 0, B_3 > 0$



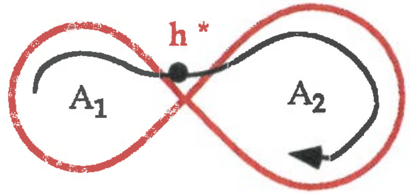
after • one gains energy
and climbs the hill

A_1 and A_2 shrink A_3 grows.

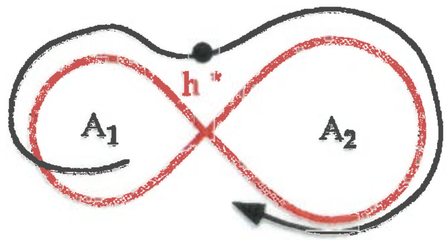
Examine the various cases . . .

$B_1 > 0,\ B_2 < 0,\ B_3 > 0$

$0 < h^* < -B_2 < B_1$



$0 < -B_2 < h^* < B_1$



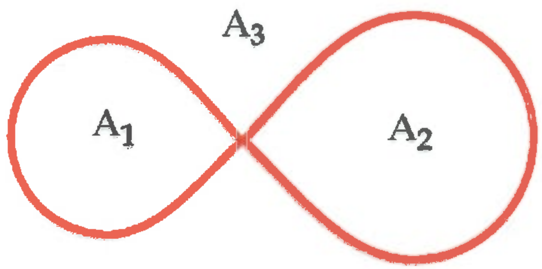
One turn around A_2 is enough to reach again a negative energy; one stays in A_2 and continue to loose energy.

One turn around A_2 is not enough to reach again a negative energy; one goes around A_1 again and gains more energy.

The phase at which the separatrix is crossed is unknown; one assumee equiprobability for the value of h^* in the interval $[0\ ,\ B_1]$

Both A_1 and A_2 grow; they attract orbits in proportion to their rate of growth.

Summary



$$\mathrm{Prob}\,(A_i \rightarrow A_j) = - \frac{\partial\, \mathrm{Area}_j \;/\; \partial\, t}{\partial\, \mathrm{Area}_i \;/\; \partial\, t} \quad + \quad O\,(\varepsilon \log \varepsilon^{-1})$$

When Prob < 0 , jump impossible;

When Prob > 1 , jump certain.

Jump in Area after leaving the uncertainty zone = Jump in critical areas

$+\ \varepsilon \log \varepsilon^{-1}$ F (phase of transition)

Pendulum

$$h = \frac{1}{2}(I - c)^2 + 2b\sin^2\frac{\phi}{2} = \frac{1}{2}(I - c)^2 - b\cos\phi + b$$

where b and c are slow functions of the time.
The area enclosed by a level curve :

$$\begin{aligned} 2\pi J &= \oint I\, d\phi \text{ for librations} \\ 2\pi J &= \oint I\, d\phi \ \pm 2\pi I_0 \text{ for circulations} \end{aligned}$$

Canonical transform : $(I, \phi) \rightsquigarrow (J, \psi)$ (Action - angle) depends on time.

Pendulum

In case of libration
$\alpha = h/2b < 1$ $\sin\phi/2 = \sqrt{\alpha}\ \sin\ell$
$S = 4\sqrt{b}\,\{(\alpha-1)\,\mathbf{F}(\ell,\alpha) + \mathbf{E}(\ell,\alpha)\} + c\,\phi$ $\psi = \mathbf{F}(\ell,\alpha)\ \pi/2\,\mathbf{K}(\alpha)$ $J = 8\sqrt{b}\,\{(\alpha-1)\mathbf{K}(\alpha) + \mathbf{E}(\alpha)\}/\pi$ $\frac{\partial J}{\partial h} = 2\sqrt{b}\,\mathbf{K}(\alpha)/\pi$

Pendulum

In case of circulation
$\beta^{-1} = h/2b > 1$ $\sin\phi/2 = \sin\theta$
$S = 4\sqrt{b/\beta}\,\mathbf{E}(\theta,\beta)\ \text{sign}\,(I-c) + c\,\phi$ $\psi = \mathbf{F}(\theta,\beta)(\pi/\mathbf{K}(\beta))\ \text{sign}\,(I-c)$ $J = \frac{4}{\pi}\sqrt{b/\beta}\,\mathbf{E}(\beta) + c\ \text{sign}\,(I-c)$ $\frac{\partial J}{\partial h} = \sqrt{\beta/b}\,\mathbf{K}(\beta)/\pi$

The functions $\mathbf{F}(\ell,\alpha)$, $\mathbf{E}(\ell,\alpha)$ are the usual elliptic integrals.

The remainder function of the canonical transformation going from (ϕ, I) to (ψ, J) :

Libration :

$$\epsilon R = \dot{c}\,\phi + \frac{2\,\dot{b}}{\sqrt{b}}\,\mathbf{Z}(\ell, \alpha)$$

Circulation :

$$\epsilon R = \frac{\dot{c}}{\mathbf{K}(\beta)}\,\{2\,\mathbf{K}(\beta)\,\theta - \pi\,\mathbf{F}(\theta, \beta)\} + \frac{2\,\dot{b}}{b}\,\sqrt{b/\beta}\,\mathbf{Z}(\theta, \beta)$$

where **Z** is the Jacobi's function.

In both cases, the mean value of the remainder function vanishes

$$< R >= \frac{1}{2\pi} \int_0^{2\pi} R \, d\psi = 0$$

First order correction to the adiabatic invariant:

Libration

$$\bar{\mathrm{J}} = J + \frac{2}{\pi} \, \mathbf{K}(\alpha) \, \{ \frac{2 \, \dot{b}}{b} \, \mathbf{Z}(\ell, \alpha) + \frac{\dot{c}}{\sqrt{b}} \, \phi \} + 0 \, (\frac{\epsilon^2 \log^2(\alpha - 1)}{\alpha - 1})$$

Circulation

$$\begin{aligned} \bar{\mathrm{J}} &= J + \frac{1}{\pi} \, \mathbf{K}(\beta) \, \{ \frac{2 \, \dot{b}}{b} \mathbf{Z}(\theta, \beta) + 2 \, \sqrt{\beta / b} \, \dot{c} \, \theta \} - \dot{c} \sqrt{\beta / b} \, \mathbf{F}(\theta, \beta) \\ &+ 0 \, (\frac{\epsilon^2 \log^2(\beta - 1)}{\beta - 1}) \end{aligned}$$

$$J_1^\star = \frac{4}{\pi}\sqrt{b} + c$$
$$J_2^\star = \frac{4}{\pi}\sqrt{b} - c$$
$$J_3^\star = \frac{8}{\pi}\sqrt{b}$$

with $h_1^\star = h_2^\star = h_3^\star = 32\ b$, $g_1 = g_2 = g_3 = 0$ and $\omega = \sqrt{b}$

Two parameters $b = b(t)$ and $c = c(t)$

$$B = \frac{2}{\pi}\frac{\dot{b}}{\sqrt{b}} \quad \text{and} \quad C = \frac{\pi}{2}\sqrt{b}\frac{\dot{c}}{\dot{b}}$$

The derivatives of the critical areas:

$$\epsilon\frac{\partial J_1^\star}{\partial\lambda} = B(1+C) \quad \epsilon\frac{\partial J_2^\star}{\partial\lambda} = B(1-C) \quad \epsilon\frac{\partial J_3^\star}{\partial\lambda} = 2B$$

The probabilities of transition:

$$FPr(i,j) = -\,\text{sign}\,(h_i h_j)\frac{\partial J_j^\star}{\partial\lambda} \;/\; \frac{\partial J_i^\star}{\partial\lambda}$$

Possible motion from $i \rightsquigarrow j$:

Confirmed Capture	if $FPr(i,j) \geq 1$
Potential Capture with probability $FPr(i,j)$	if $0 \leq FPr(i,j) \leq 1$
Escape	if $FPr(i,j) \leq 0$

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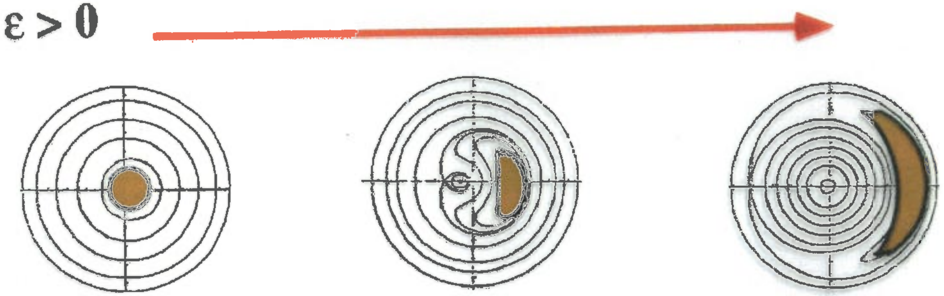
Resonance sweeping

$$\ddot{\theta} + \sin\theta = \sin[1 - \varepsilon t]\, t$$

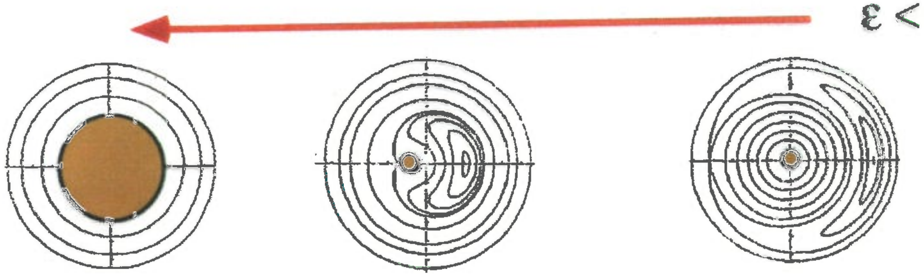
$$H = -3\,\lambda(\varepsilon t)\,R + R^2 - 2\sqrt{2R}\cos r$$

$\varepsilon > 0$



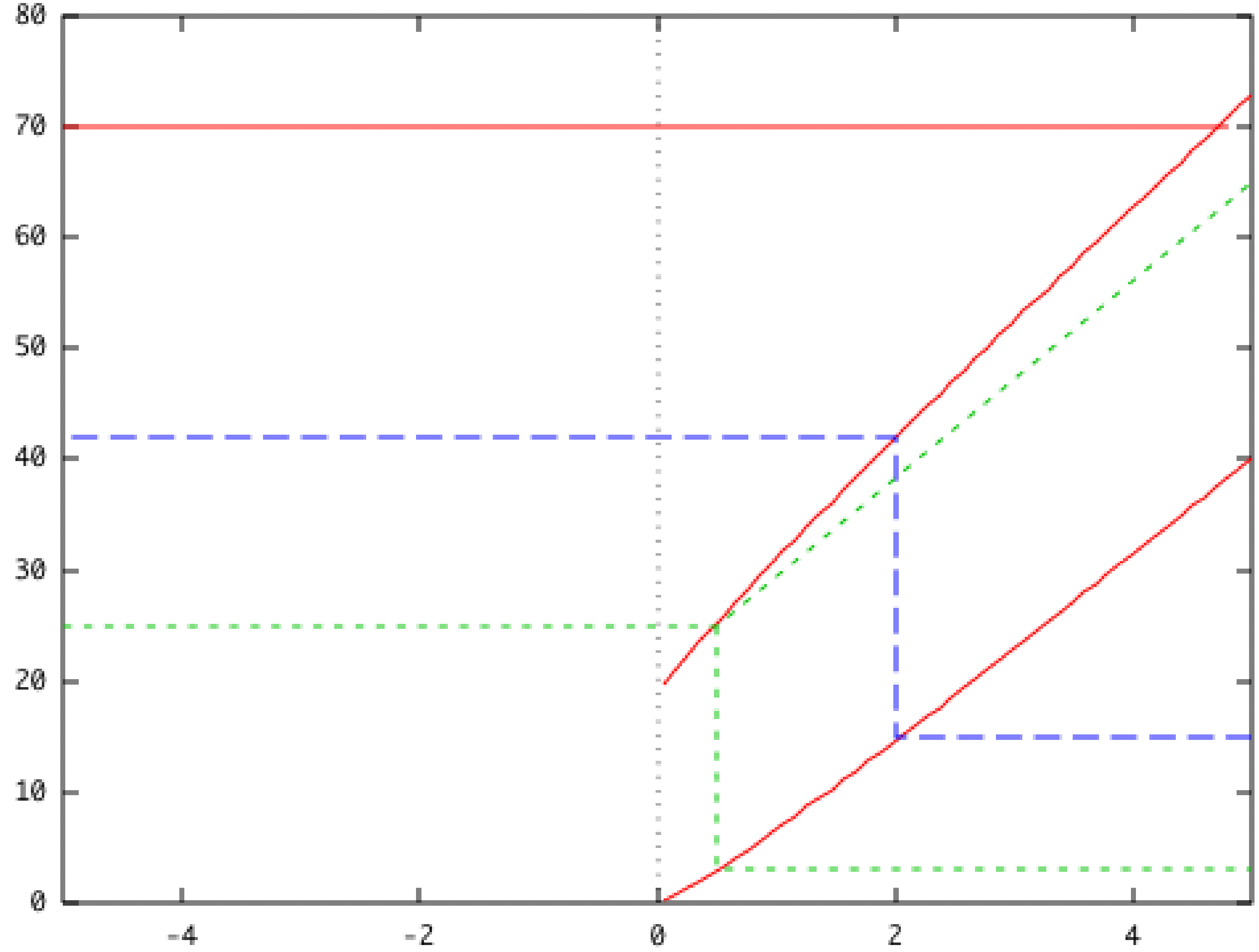
forced oscillation of increasing amplitude

$\varepsilon < 0$



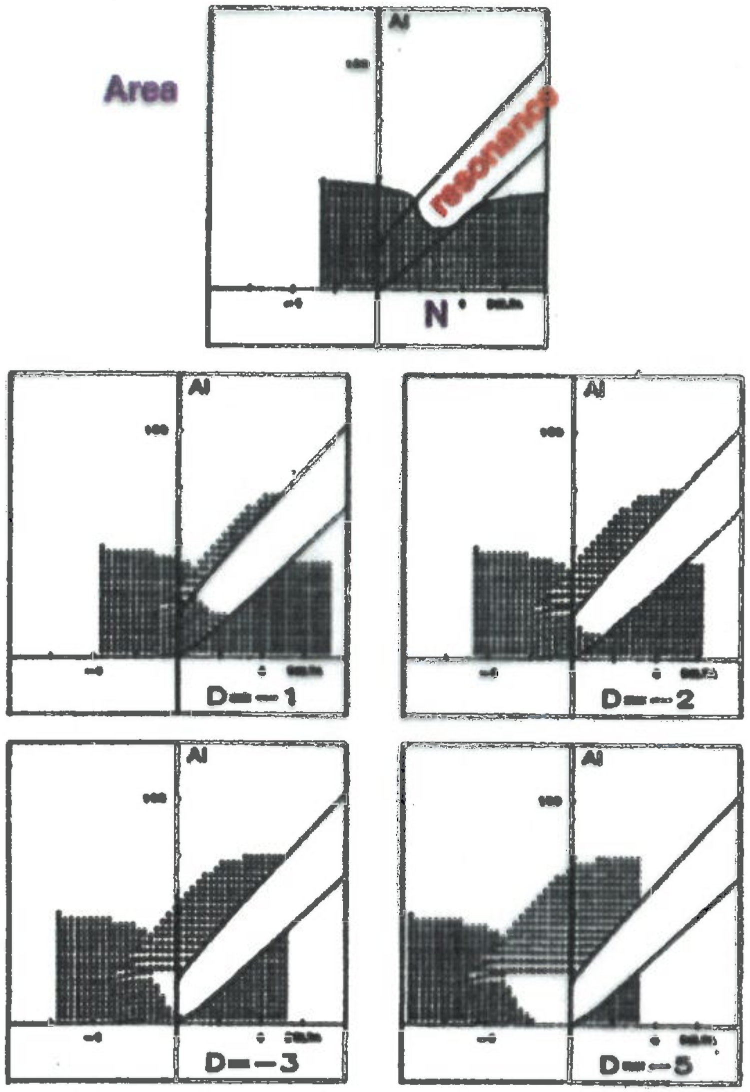
Lun. 14-bis

free oscillation of constant amplitude

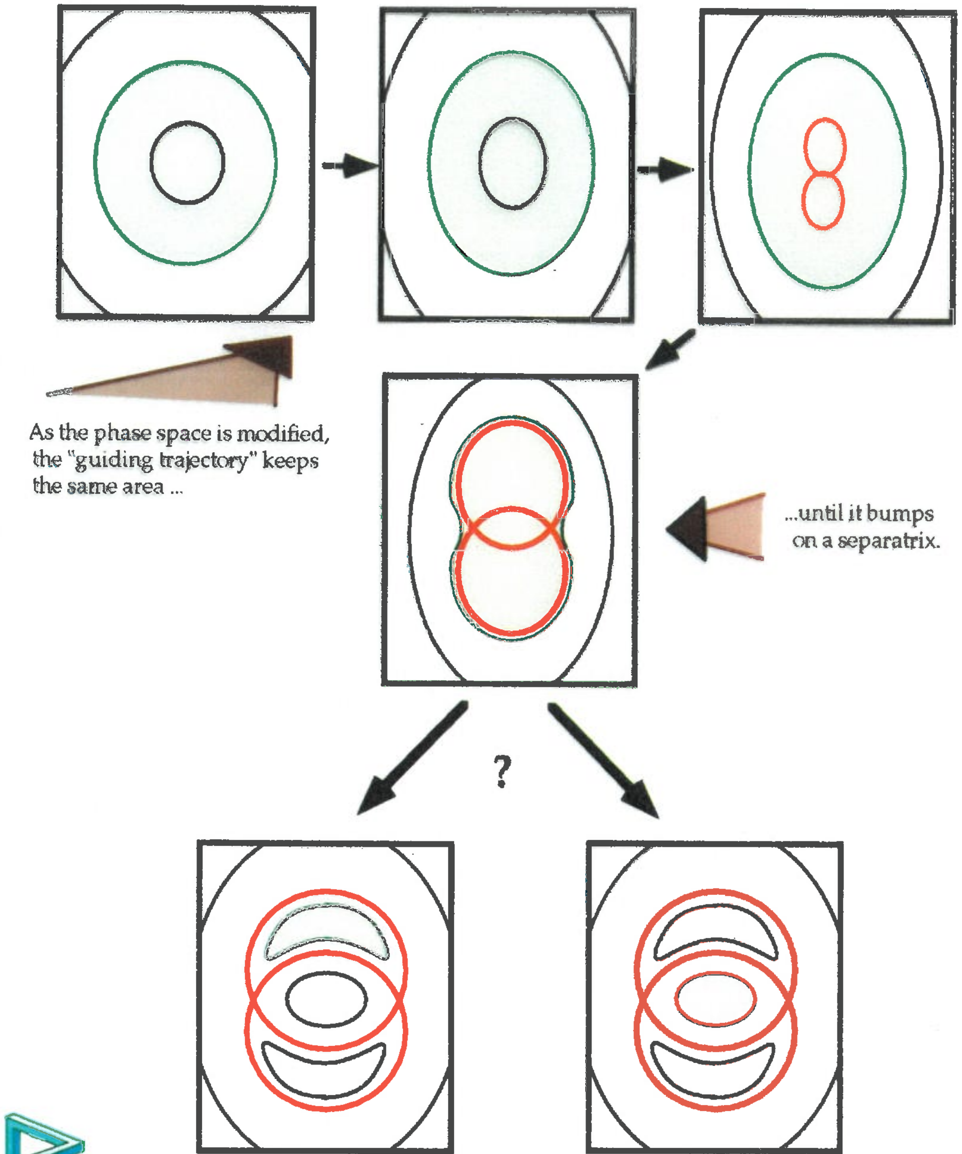

80
70
60
50
40
30
20
10
0
-4
-2
0
2
4

Resonance sweeping

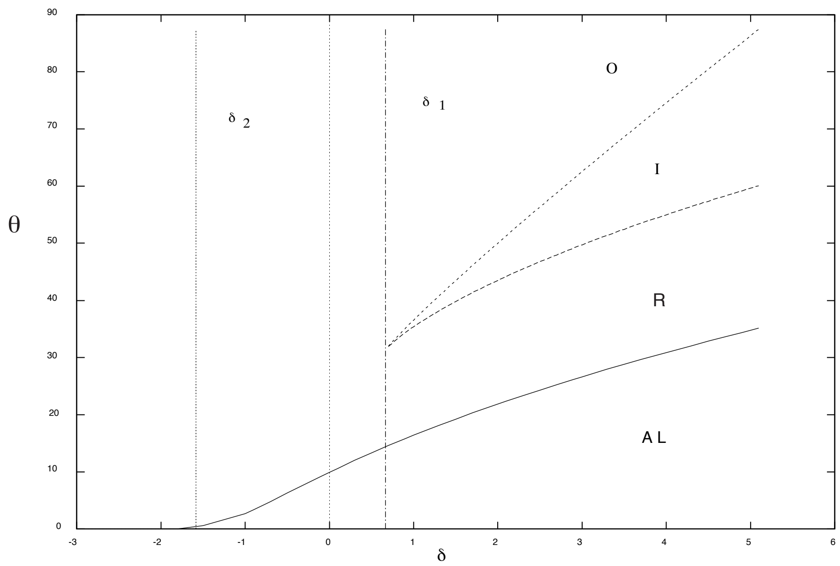
Lemaître 1984



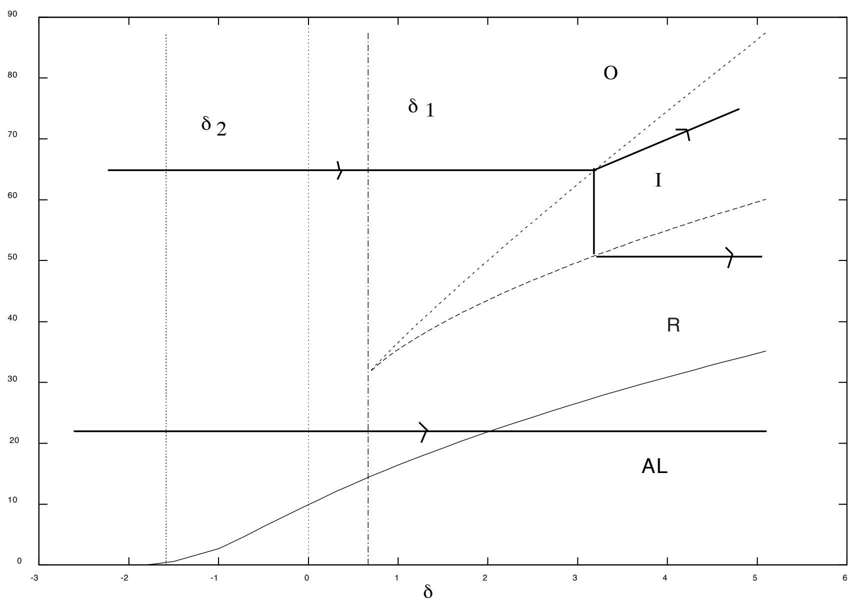
Separatrix crossing



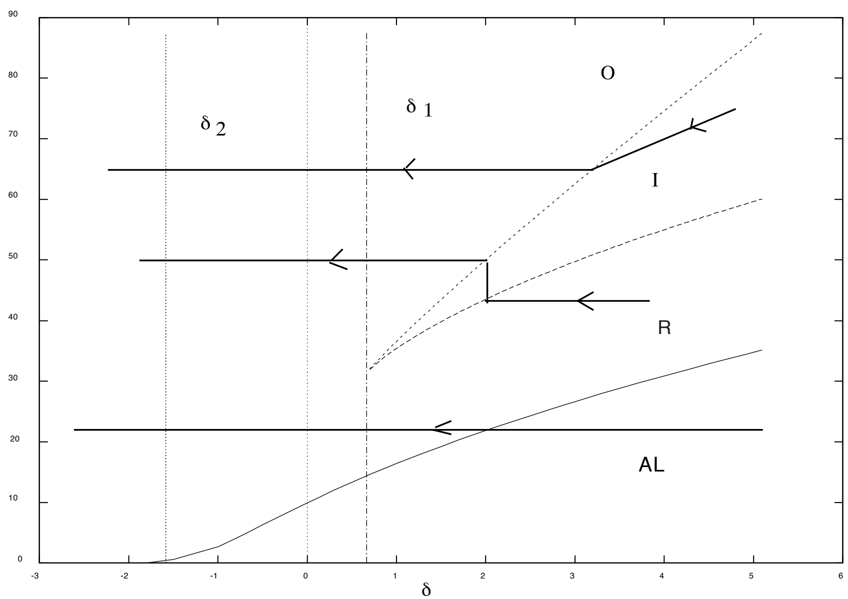
b=1



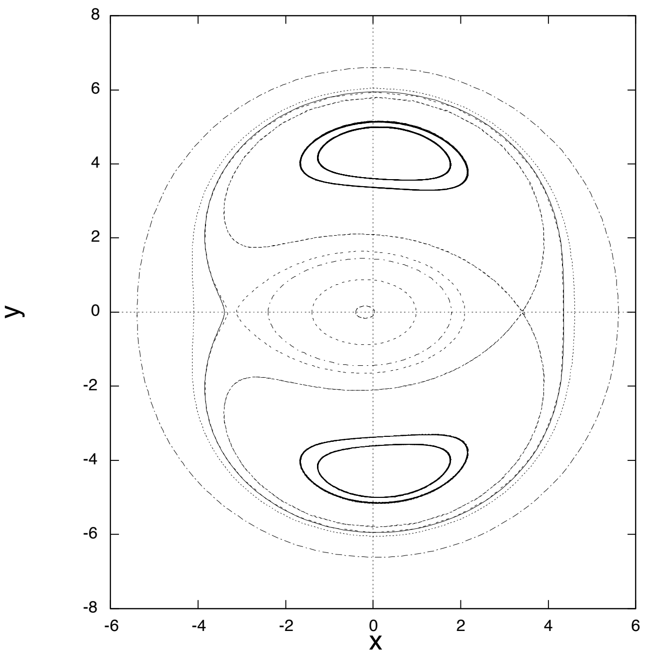
θ
δ
δ_2
δ_1
O
I
R
A L

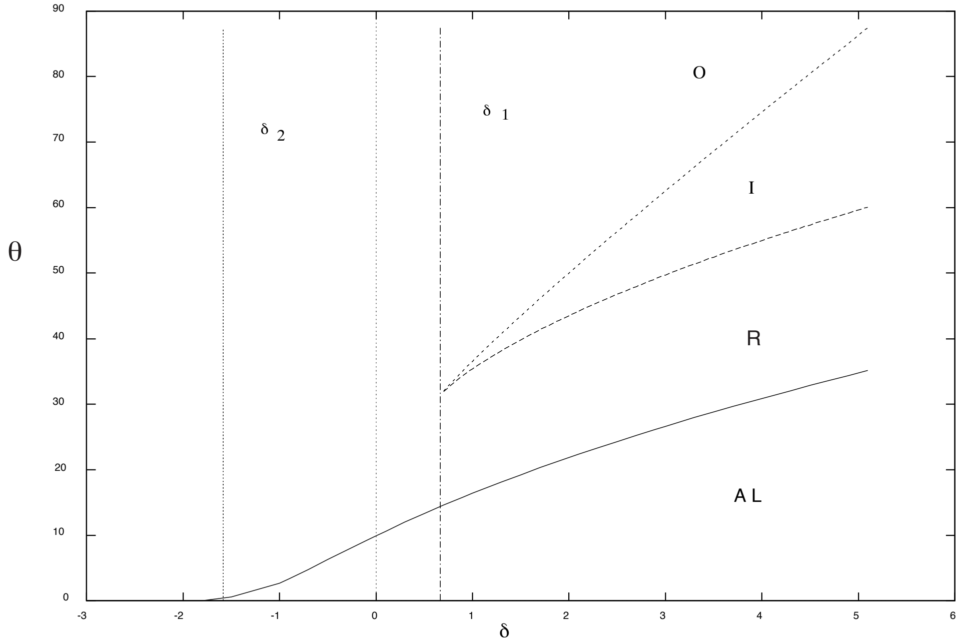


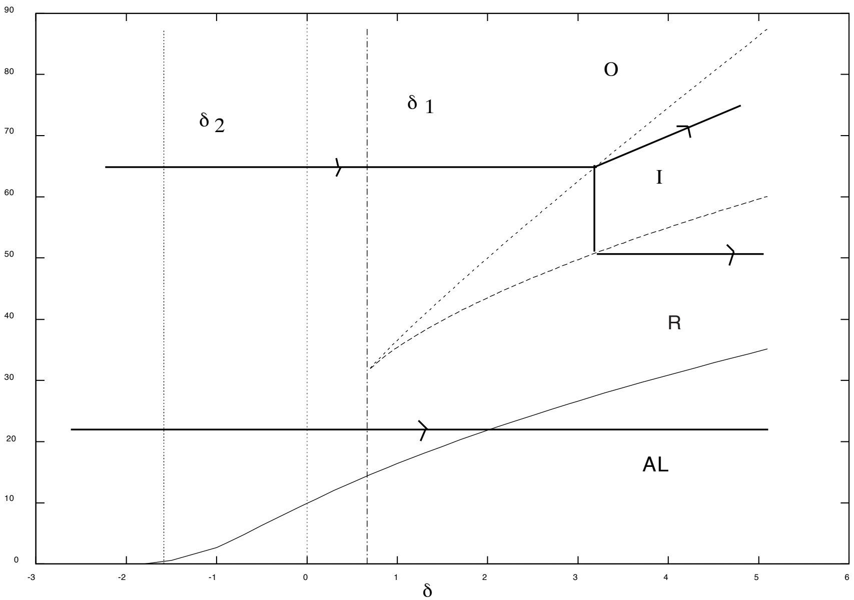
δ
δ_2
δ_1
O
I
R
AL

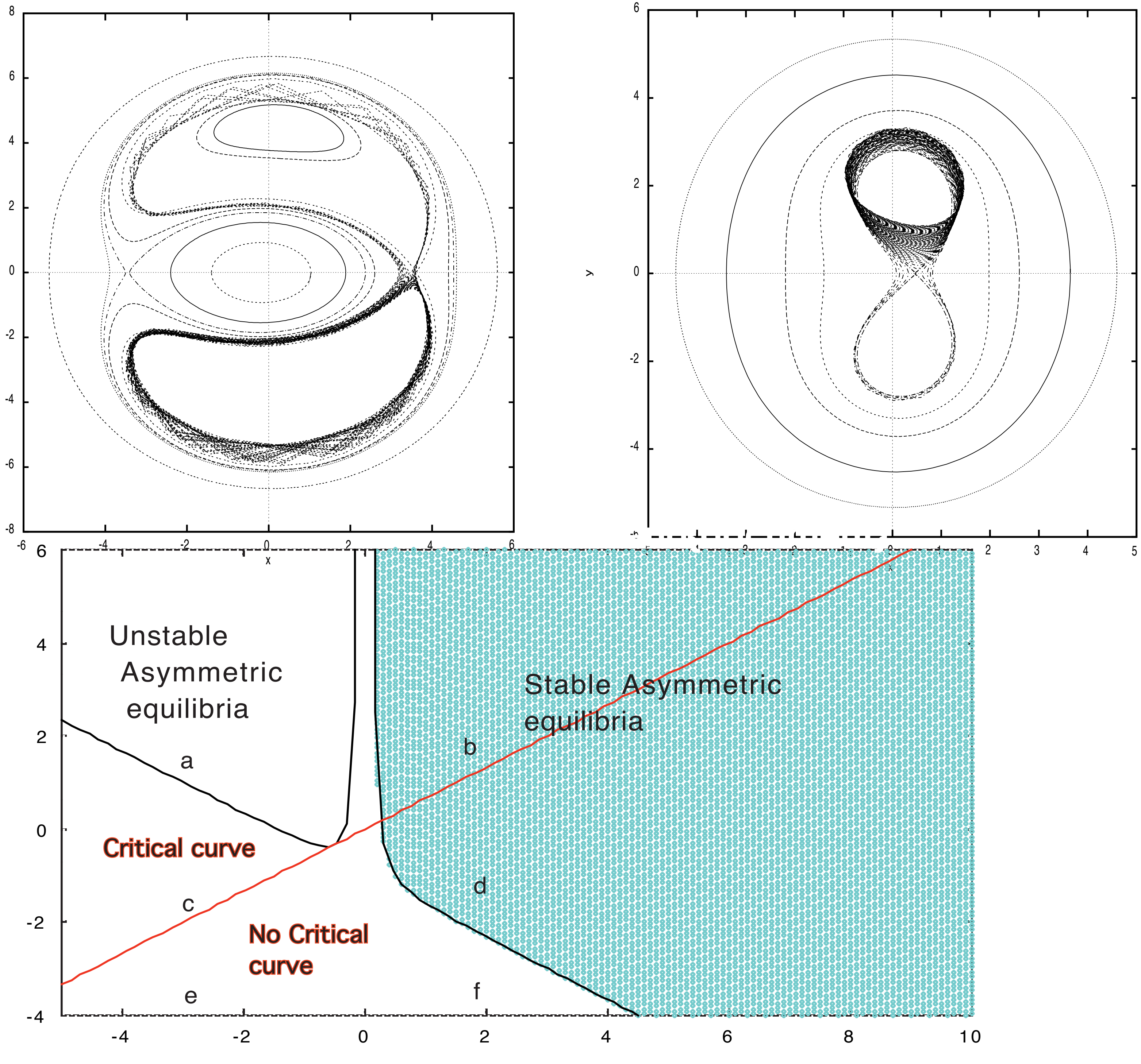


δ
δ_2
δ_1
O
I
R
AL


y
x


θ
δ
δ 2
δ 1
O
I
R
A L


δ
δ 2
δ 1
O
I
R
AL



Unstable Asymmetric equilibria
Stable Asymmetric equilibria
a
b
Critical curve
c
d
No Critical curve
e
f
x
y

Traversée et capture en résonance

Variation du demi-grand axe

Forces non conservatives :
Poynting-Robertson drag
Stockes drag
Dissipation de la nébuleuse
Effet Yarkovsky

Vide d'une résonance | Capture en résonance | Saut d'une résonance

Ephemerides for the asteroids

Anne Lemaitre
University of Namur
Belgium

UNIVERSITY OF NAMUR

Long term evolution

- 2-body problem - 3-body problem
- N-body problem
- Resonances : mean motion, secular
- Gravity + collisions
- Families - proper elements
- Chaos and stable chaos
- Secular non gravitational drags
- Yarkovsky effect
- YORP Rotation and Spin
- Hundreds of millions of years

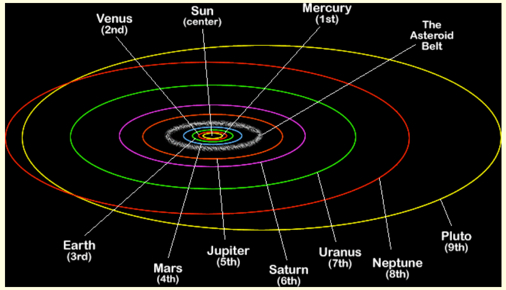
Population

- Minor Planet Center : catalogues
- Non numbered asteroids
- Systematic Surveys : magnitude 22
- 2010 : 98 % of asteroids of 1 km
- Masses, spin velocities, densities
- Albedos, thermal conductivity, surface temperatures
- Irregular shapes - binaries
- Planetary ephemerides Standish &Fienga, 2002

Short timescales

- Non gravitational effects
  - Direct radiation pressure
  - Thermal forces
  - Yarkovsky effect
- 2 sources :
  - Bottke et al, 2006 : Yarkovsky and YORP effects: implications for the asteroids
  - Milani & Gronchi 2008 (ch 14) : non gravitational forces
- Comparisons

Classical Models

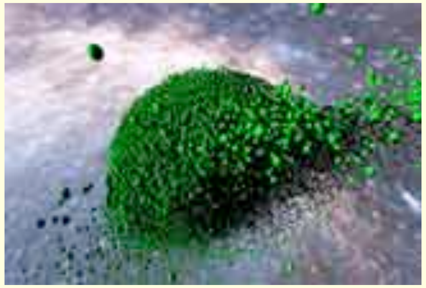


- Gravitational forces : Sun + planets
- Main belt : 10^6 objects with D> 1 km
- Collisions (5 km/s)

→ History of the asteroid population over several billion years

orbits - collisions - craters and fragments

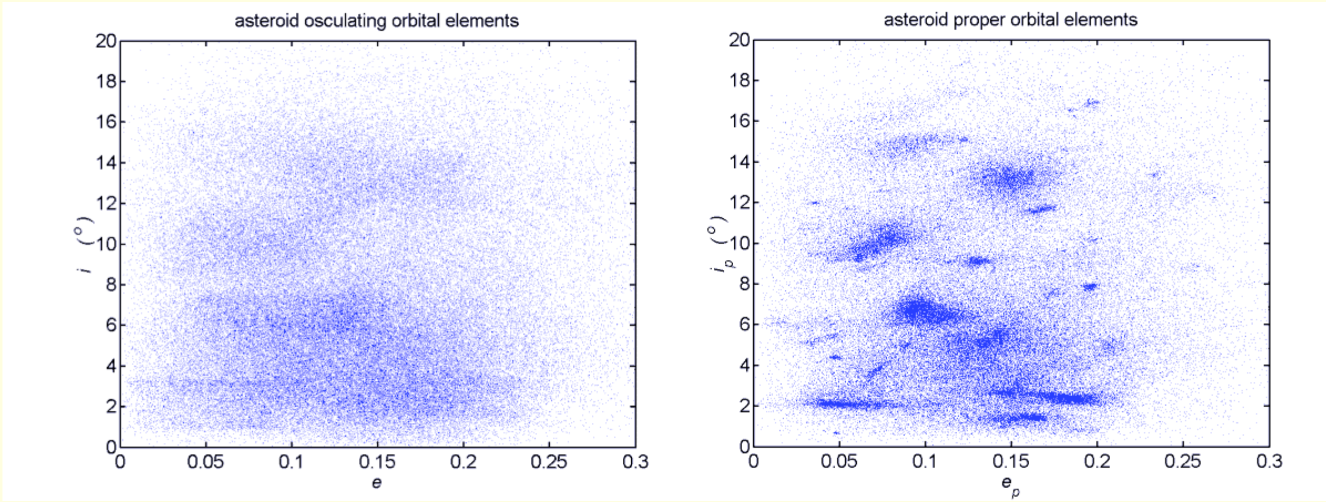
- Largest impacts : origin of asteroid families (dynamical and spectral)
- Invariant of motions : representations in PROPER elements
- Ejecta in several directions and velocities, injected in resonances



Collisions

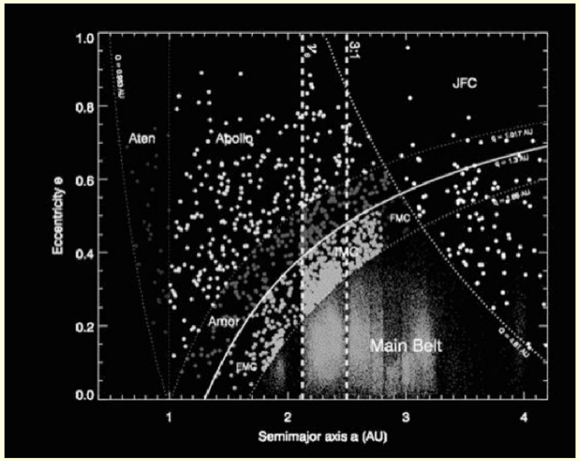
and Fragmentation

Michel & Tanga 2001



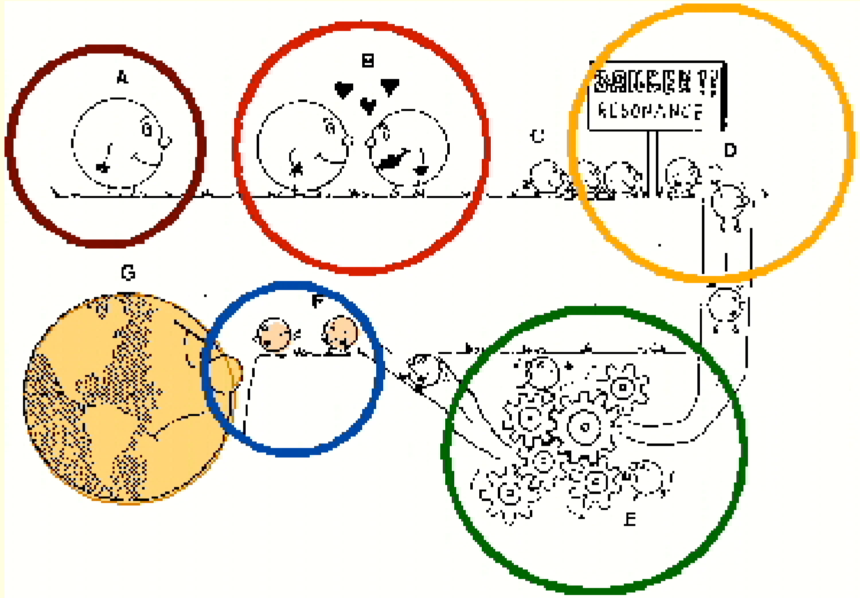
Proof : Asteroid large families

proper elements phase space



Presence of
resonances

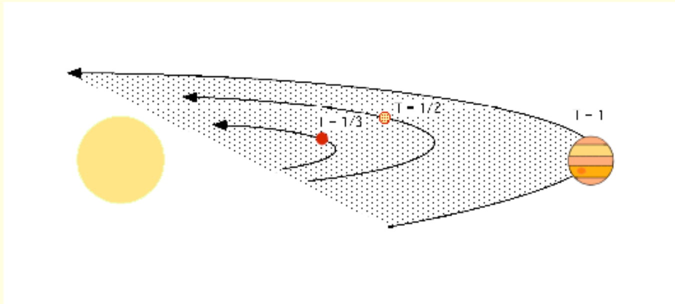
Asteroid life (Zappala)



Reproduced from: V. Zappala and A. Cellino: Celestial Mechanics, 54, 207-227, 1992.

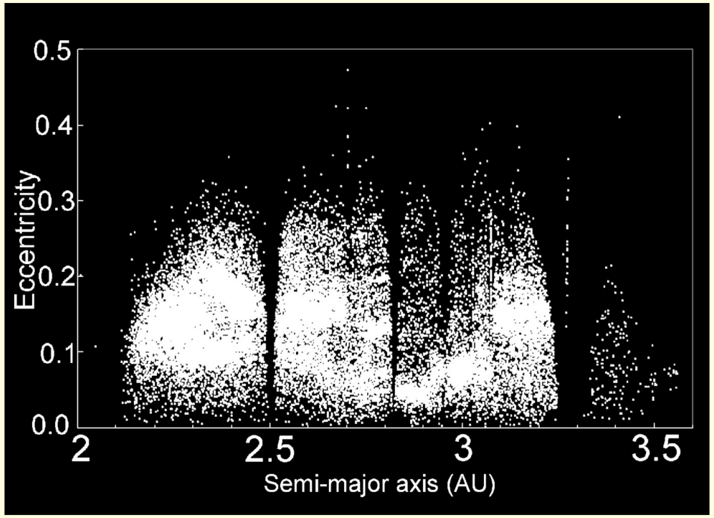
- Clues to the formation of the Solar System
- "Natural experiments" on the physics of such objects (collision - differentiation - outgasing ...)
- Studies of resonance phenomena
- "Natural experiments" on long term dynamical evolution
- Source of Near Earth Objects

Resonances

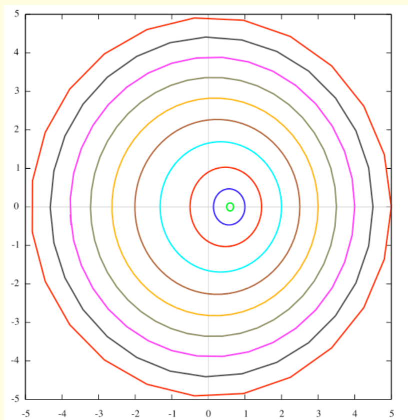


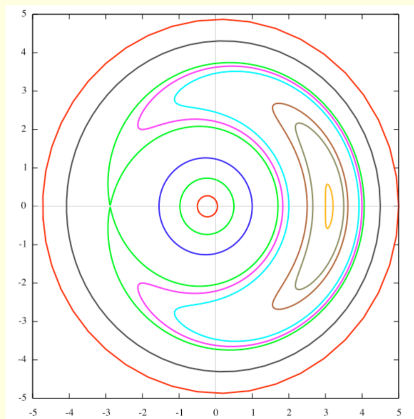
- Mean motion resonances :

  commensurability between the mean motions of an asteroid and a planet (2/1, 3/1, 5/2, 7/3)



- Semi-major axis blocked
- Large excursions in eccentricity
- Superposition of several degrees of freedom

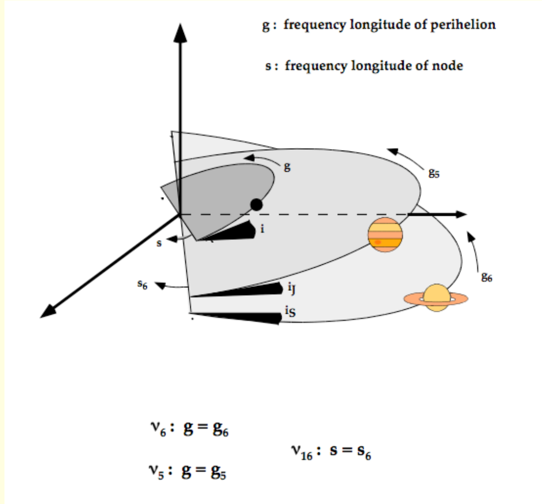


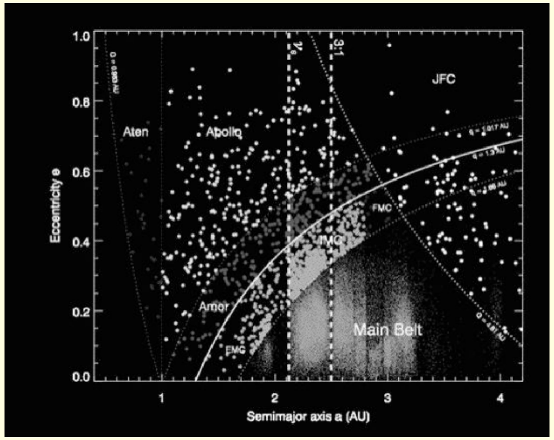


Main belt - collisions - resonances - pumping of the eccentricity - Mars or Earth crossers

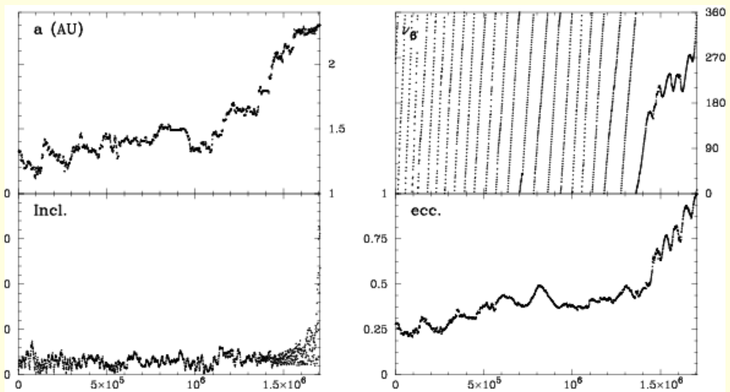
Resonances

- Secular resonances :
  commensurability between the motions of the pericenter or the node of an asteroid and this of a planet (ν_5, ν_6, ν_{16})





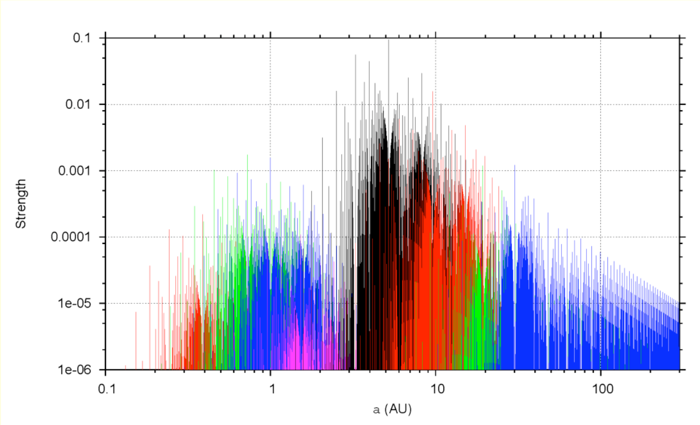
Large excursions in eccentricity and in inclination



Main belt - collisions - secular resonances - Mars or Earth crossers

Itokawa
Yoshikawa & Michel 2006

2 and 3- body resonances : chaos



$k_1\, w_1 + k_2\, w_2 = 0$

$k_1\, w_1 + k_2\, w_2 + k_3\, w_3 = 0$

Mean motion : 10 years

Secular : 10^4 years

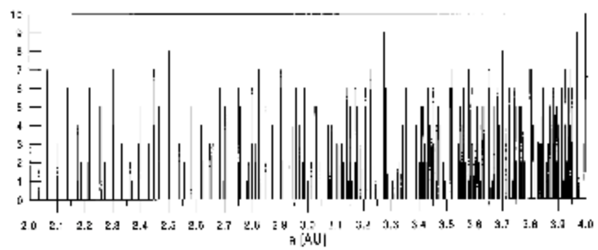


FIG. 3.—Location of the ordinary mean motion resonances (*heavy lines*) and the three-body resonances (*light lines*). The height of each bar is related to the resonance order, as explained in the text, so that the vertical scale gives a comparative indication of the strength of the resonances.

Chaos : Maximum Lyapounov exponant >0

High order resonances 12:7

Chaotic orbit (Helga 522)

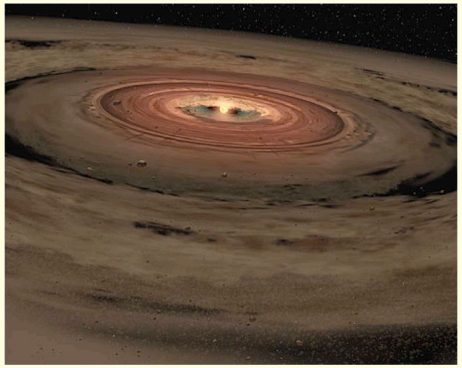
Milani, 1993

Confined (stable) chaos

Nesvorny - Morbidelli 1998

Problems with classical Models

- Observations of asteroids : sizes, ages, velocities, spin rates Bottke et al 2006
- Catastrophic events are too inefficient



→ Non gravitational forces

- Stockes drag (gas)
  - small objects moving through a fluid
  - viscous resistance to the fluid
  - proportional to the relative velocity

$$\vec{F}_{Stokes} = \frac{1}{2}\rho_{gas} V^2 A C_d \vec{V}_r$$

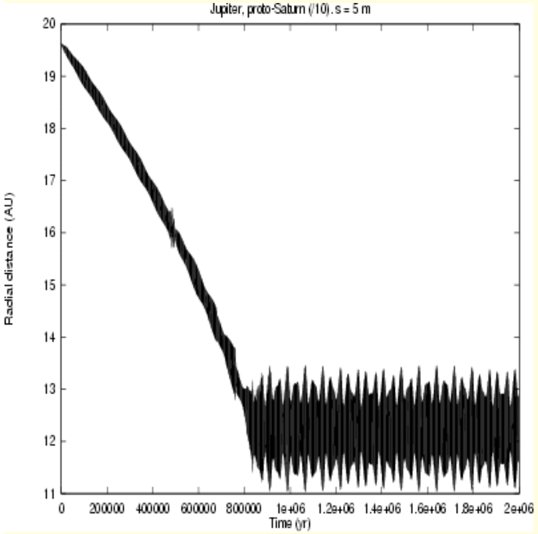
- Poynting - Robertson drag
  - tangential component of the radiation pressure
  - opposite to the orbital motion,
  - spirals into the Sun (increase orbital speed)

$$\vec{F}_{PR} = -\frac{\Phi}{c^2}\vec{V}$$

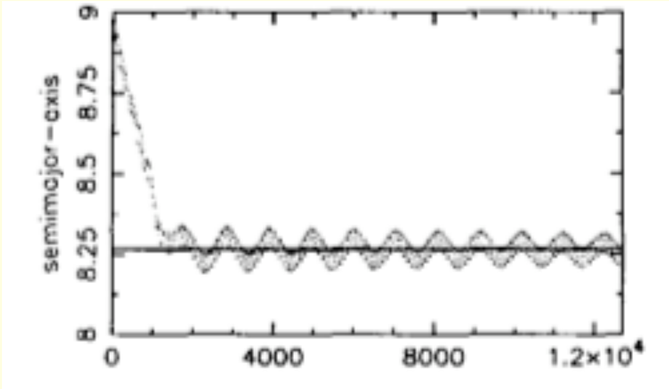
- Grains
- Dust
- Nebula or ring
- Decreasing a

$$-\vec{V}$$

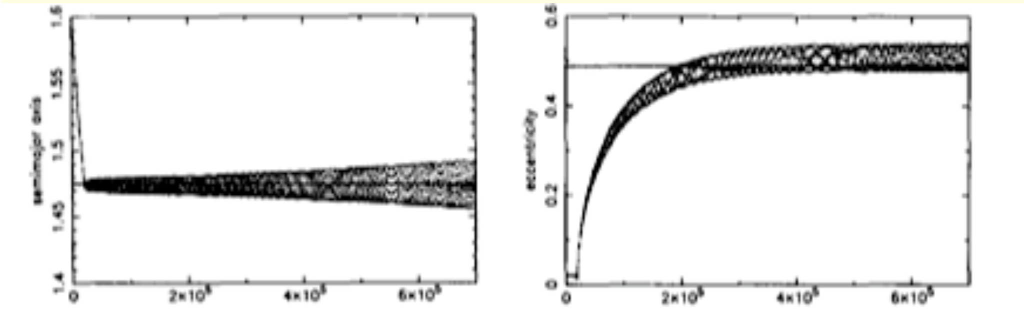
Capture of grains in external resonances



Gomes (PR drag) 1995

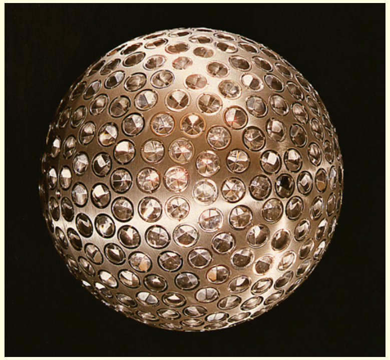


Beaugé and Ferraz-Mello (Stokes drag) 1993



Beaugé and Ferraz-Mello (PR drag) 1994

Yarkovsky secular effect



Lageos

- Thermal properties of the body
- Optical properties of the surface
- Illumination from the Sun
- Orbital dynamics

Diurnal effect

- thermal conductivity < ∞
- surface temperature not immediately increased
- time lag between maximum illumination and maximum temperature
- anisotropic emission of thermal radiation
- maximum intensity at a finite angle to the Sun’s direction
- cold morning, warm noon, hot afternoon

Seasonal effect

- some portions of the surface illuminated for longer times during any rotation (spin inclination - eccentricity)
- anisotropic emission of thermal radiation
- resultant force along the spin axis (to the “winter” hemisphere)

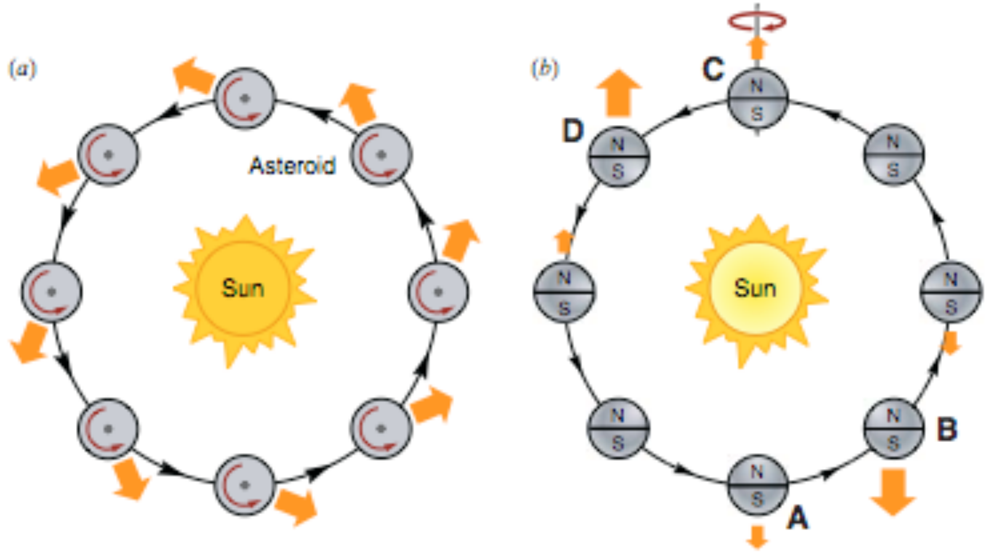


Figure 1

(*a*) The diurnal Yarkovsky effect, with the asteroid's spin axis perpendicular to the orbital plane. A fraction of the solar insolation is absorbed only to later be radiated away, yielding a net thermal force in the direction of the wide arrows. Because thermal reradiation in this example is concentrated at about 2 PM on the spinning asteroid, the radiation recoil force is always oriented at about 2 AM. Thus, the along-track component causes the object to spiral outward. Retrograde rotation would cause the orbit to spiral inward. (*b*) The seasonal Yarkovsky effect, with the asteroid's spin axis in the orbital plane. Seasonal heating and cooling of the "northern" and "southern" hemispheres give rise to a thermal force, which lies along the spin axis. The strength of the reradiation force varies along the orbit as a result of thermal inertia; even though the maximum sunlight on each hemisphere occurs as A and C, the maximum resultant radiative forces are applied to the body at B and D. The net effect over one revolution always causes the object to spiral inward.

Diurnal effect

- Spherical body
- Circular orbit
- Spin orthogonal to the orbital plane
- Prograde rotation (retrograde)
- Spiral outward (inward)
- da/dt >0 (da/dt <0)
- negligible for fast rotations

Seasonal effect

- Spherical body
- Circular orbit
- Spin in the orbital plane
- North and south hemispheres
- A and C : max illumination
- delay : thermal inertia
- B and D : max reradiation force
- globally : spiral inward
- da/dt <0

Obliquity
Spin rate
Size (D < 20 km)
Surface conductivity
Solar distance

Real situations : contributions of diurnal and seasonal aspects

Measured Yarkovsky effect

- 6489 Golevka (NEA)
- 0,5 km size
- radar observations
  in 1991, 1995, 1999, 2003
- A shift of 15 kms on 12 years
  on the semi major axis
- Chesley et al (2003)

The surface of an asteroid is heated by the Sun during its day and
cools off during its night; the asteroid emits more heat from its afternoon sight.
This unbalanced thermal radiation produces a tiny acceleration.

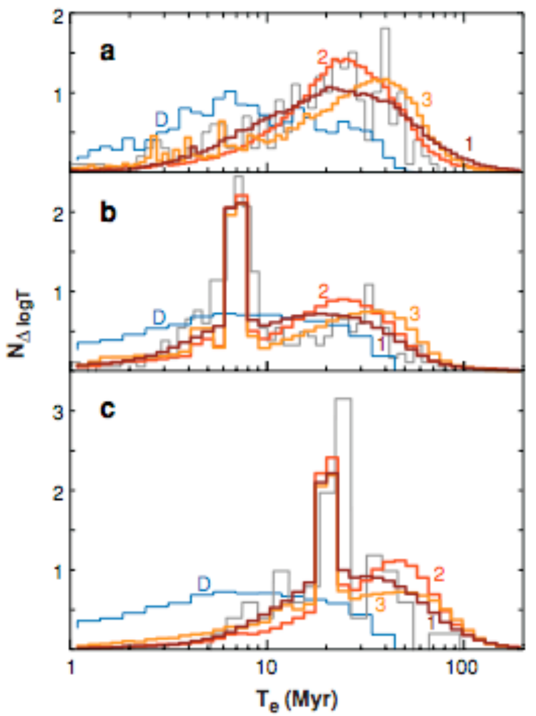
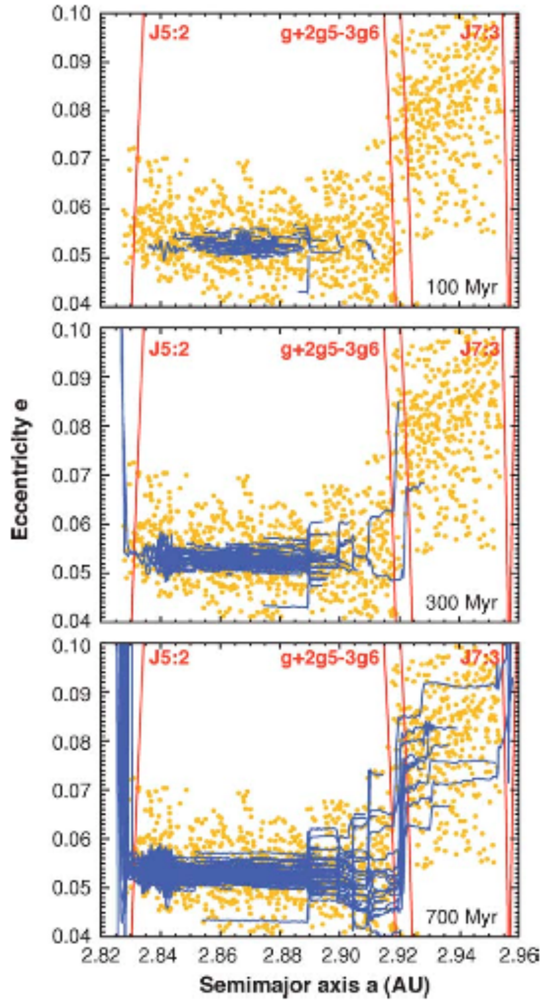


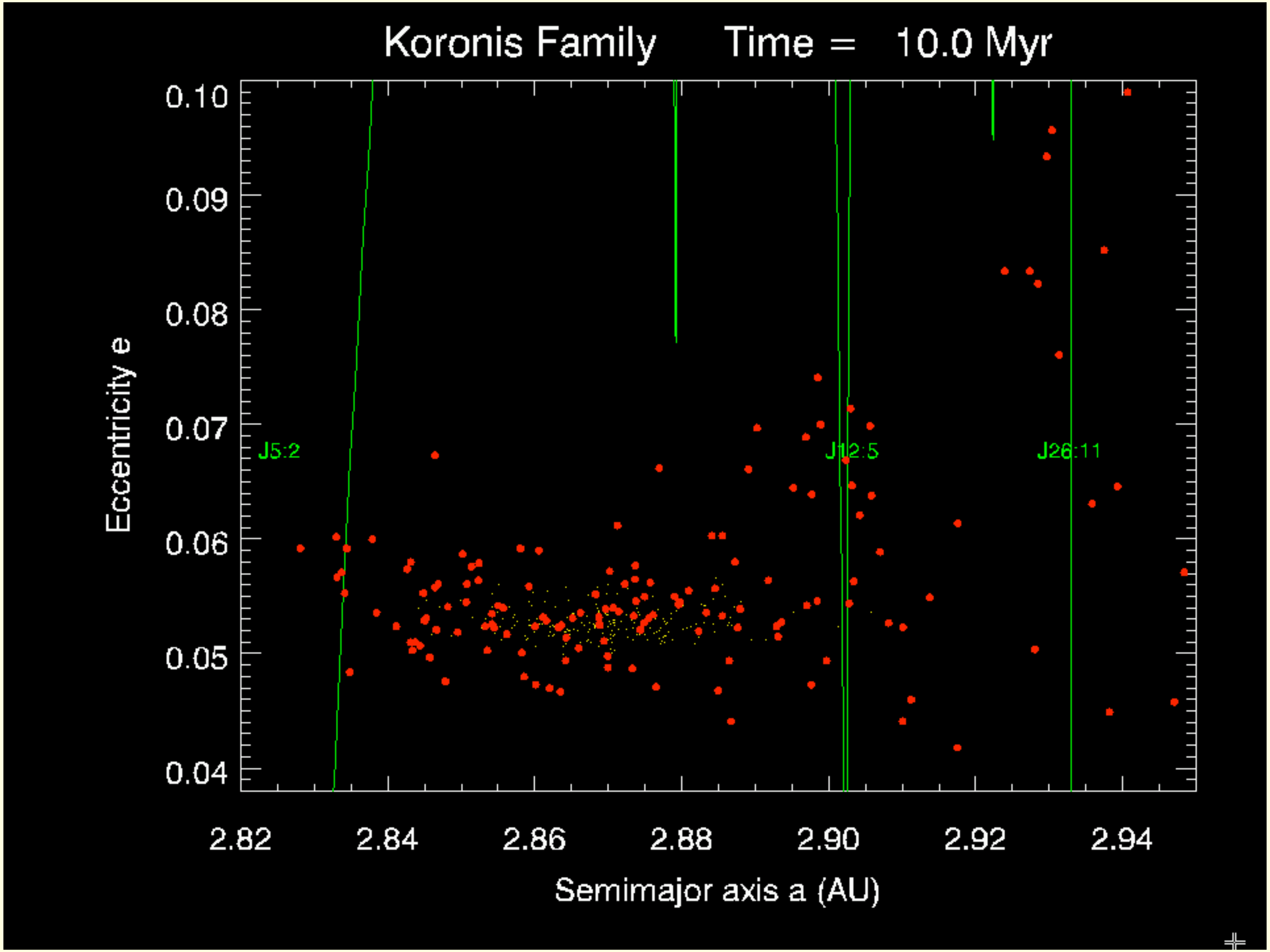
Figure 3

Comparison of the modeled and observed CRE age distributions for three different meteorite types (data—*gray histograms*). We show results of the direct-injection scenario with no Yarkovsky mobility (D histogram, *blue*) and the model including Yarkovsky mobility of the meteoroids and their precursors (*bold full-line histograms, orange and red*). Histograms 1, 2, and 3 refer to thermal conductivity values of 0.0015, 0.1, and 1 W m^{-1} K^{-1}, respectively. Part (*a*) assumes ejecta from asteroid Flora whose computed CRE ages are compared with the observed distribution for 240 L-chondrites. Part (*b*) assumes ejecta from asteroid (6) Hebe and the comparison with 444 CRE ages of H-chondrites. Part (*c*) assumes ejecta from asteroid (4) Vesta, compared to the CRE age data for 64 HED (howardite-eucrite-diogenite) meteorites. In all cases, the intermediate *K* value appears to provide the best match to the data. Note that the direct injection scenario would always predict many more short CRE ages than are observed, and a shortage of ages between 20 and 50 My, which is not observed.



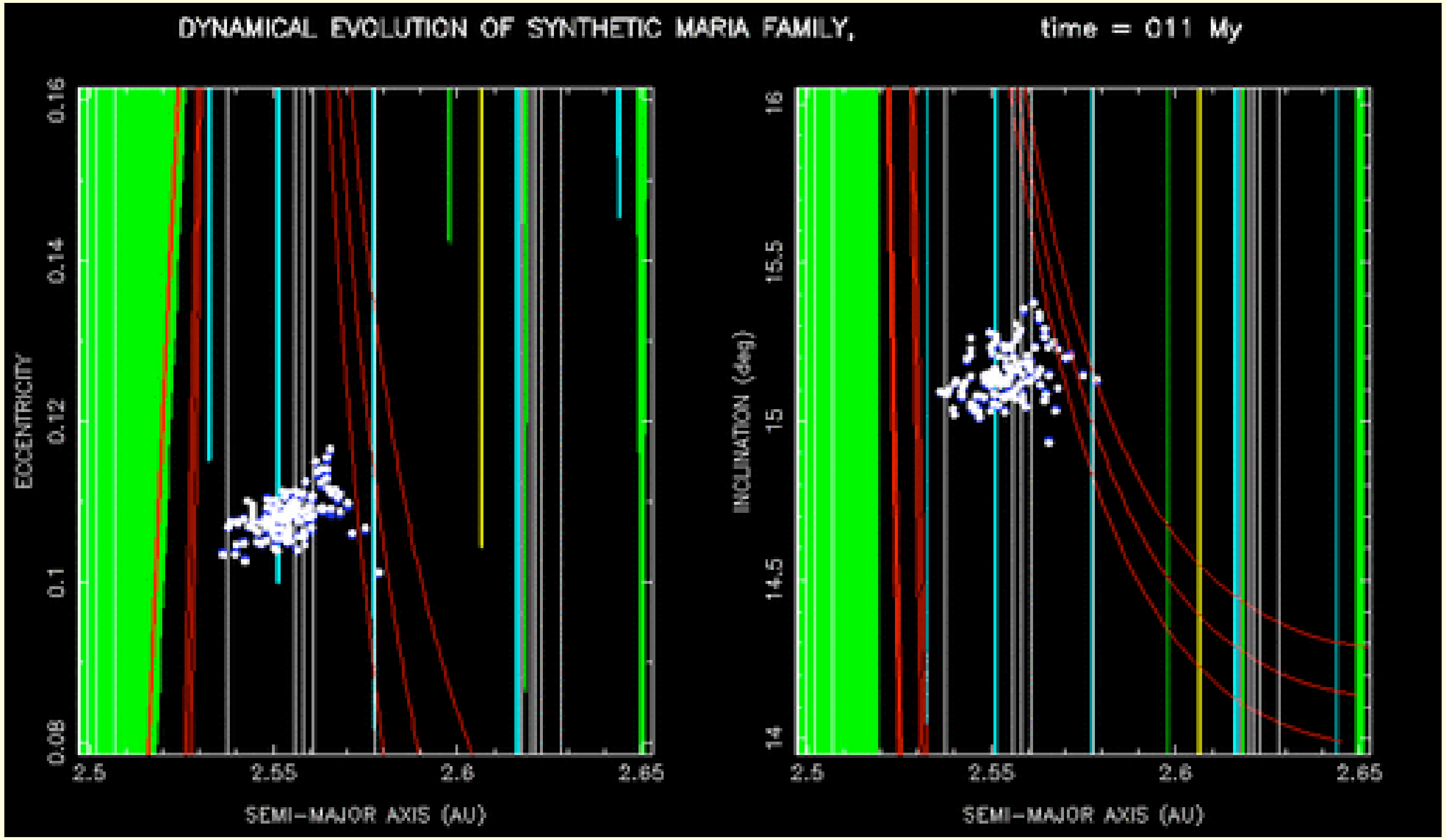
Age distributions for 3 types of meteorites
Data histograms
Classical models
Including Yarkovsky (3 thermal conductivity)

Simulation of Koronis family
Initial distribution + Yarkovsky
Fragments of different sizes
Different trappings and histories

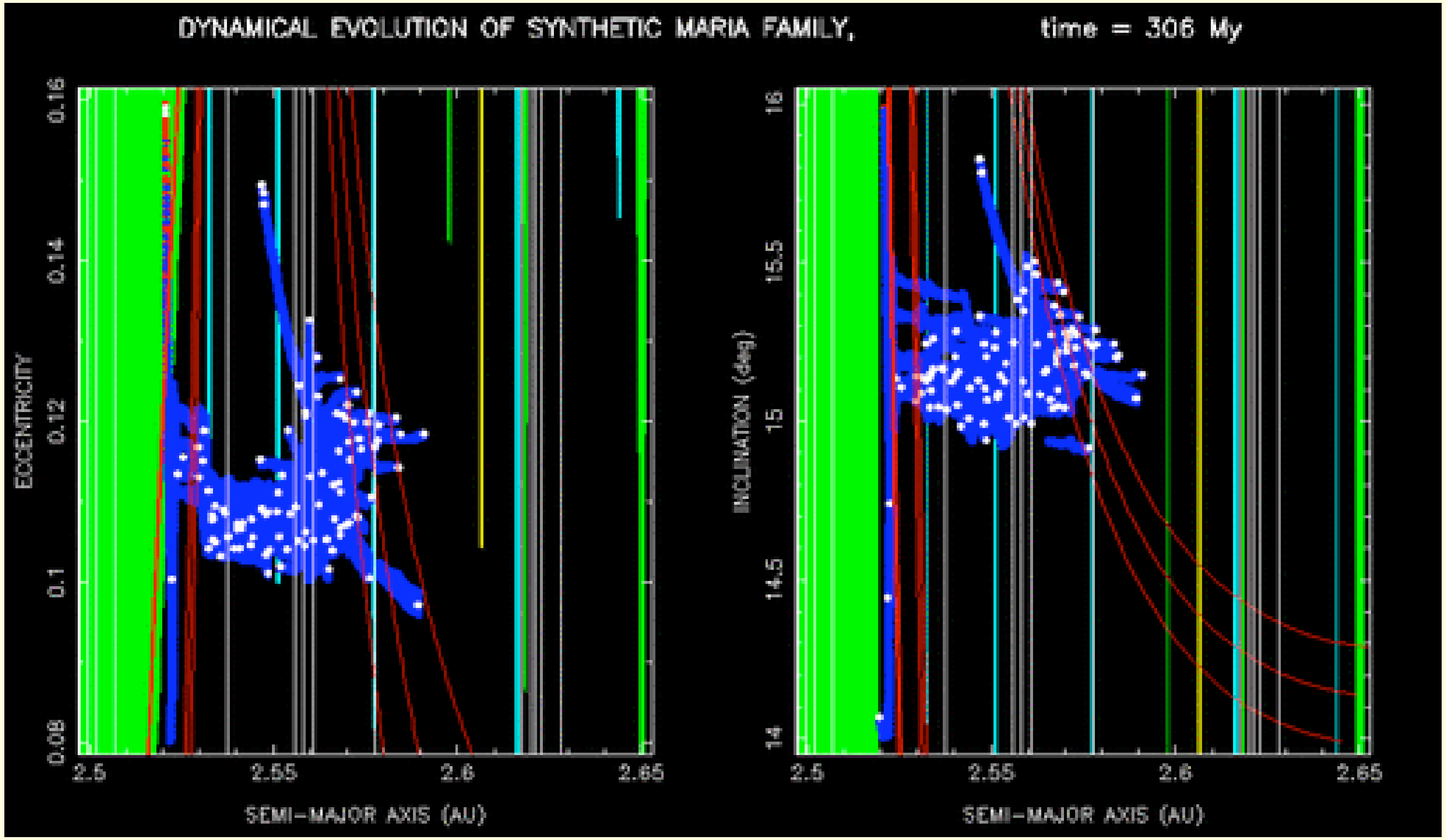

Koronis Family Time = 10.0 Myr
Eccentricity e
0.10
0.09
0.08
0.07
0.06
0.05
0.04
J5:2
J12:5
J26:11
2.82
2.84
2.86
2.88
2.90
2.92
2.94
Semimajor axis a (AU)

Bottke 2001

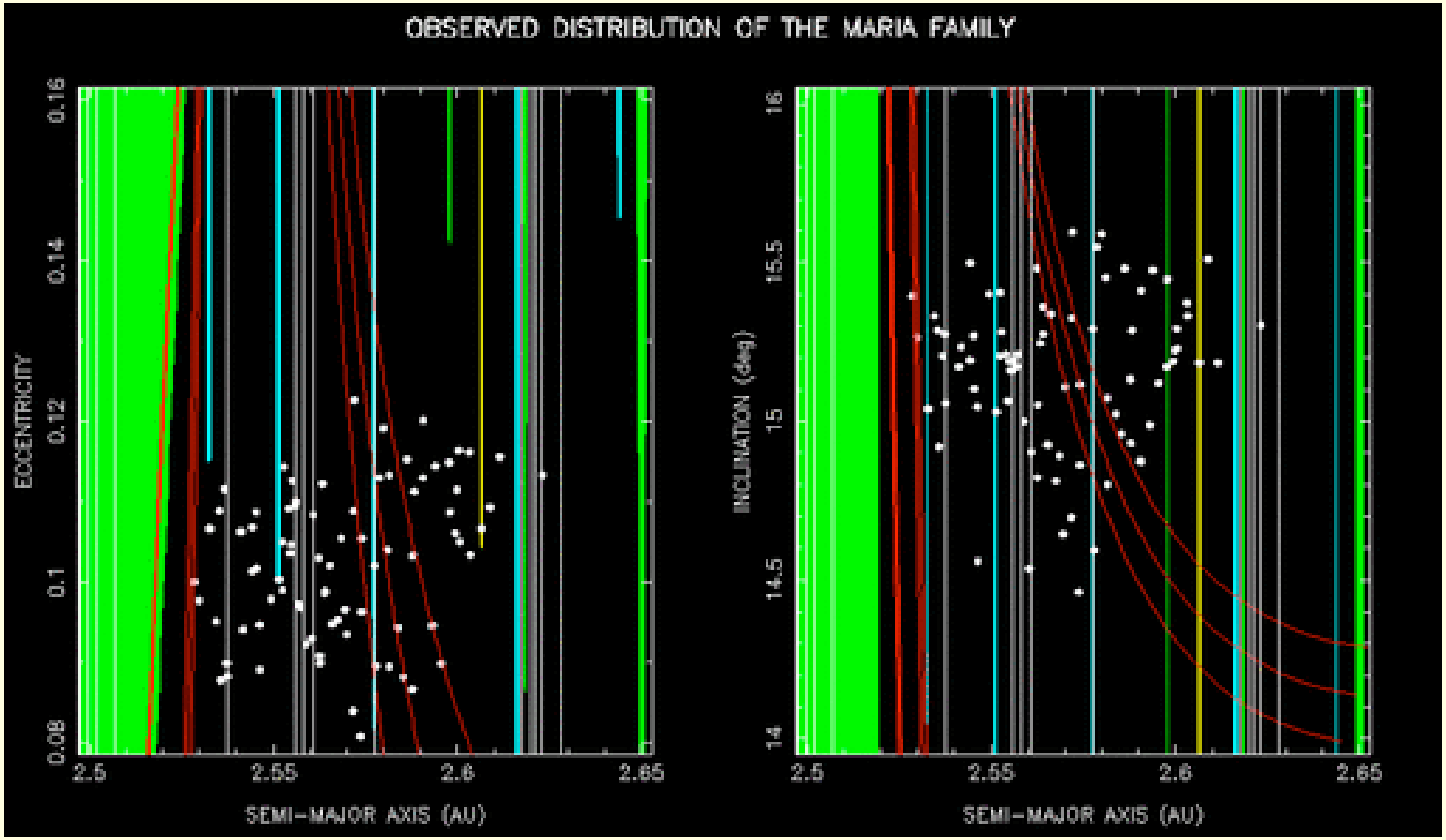
Maria Family



Maria Family



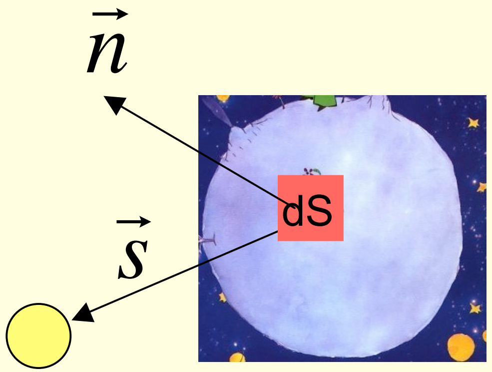
Maria Family

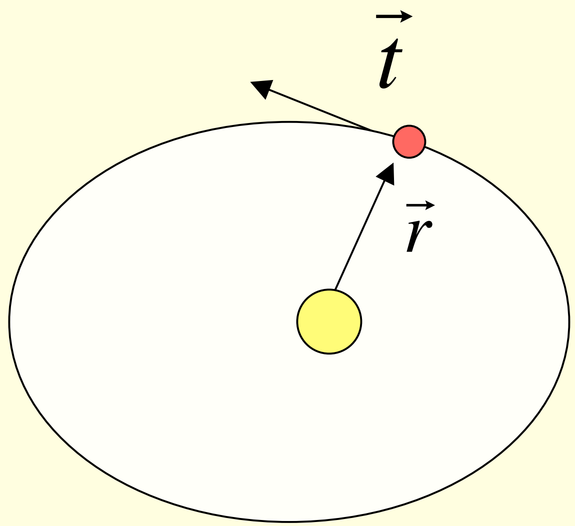


- Yarkovsky effect efficiency proved on hundreds of millions of years
- Other non gravitational forces ?
- Importance of non gravitational forces on short timescales ?

Direct radiation pressure
Thermal emission

Orbital coordinates





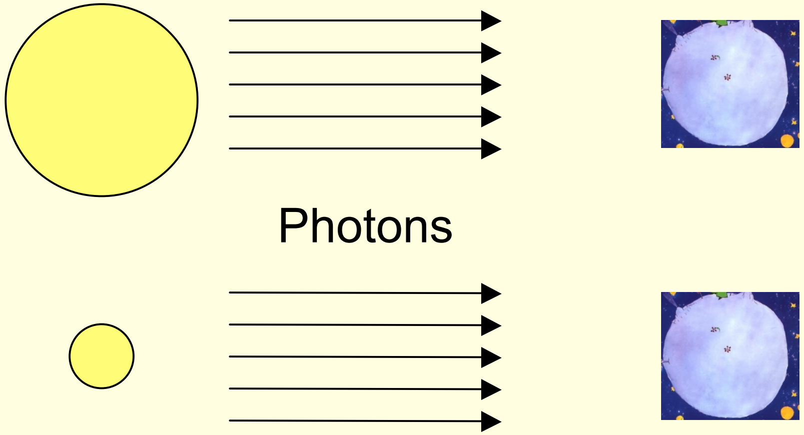
Local surface coordinates

$$\vec{s} \bullet \vec{n} = \cos\beta$$

$$(\vec{u} = \frac{\vec{r}}{r},\ \vec{t} = \vec{w} \times \vec{u},\ \vec{w} = \frac{\vec{l}}{l} = \frac{\vec{r} \times \vec{v}}{l})$$

$$r >> R_A : \vec{r} \cong -\vec{s}$$

Direct radiation pressure



Photons =
Energy + Linear
Momentum

Sun = point mass

Energy flux (per unit area) → Φ_S

Linear momentum flux (per unit area) → Φ_S / c

Transfer of linear momentum from the photons to the surface = direct radiation pressure

c = speed of light

3 mechanisms : $\alpha + \rho + \delta = 1$

Photons

Absorption (black body) : α

Reflection (mirror) : ρ

Diffusion (reemission) : δ

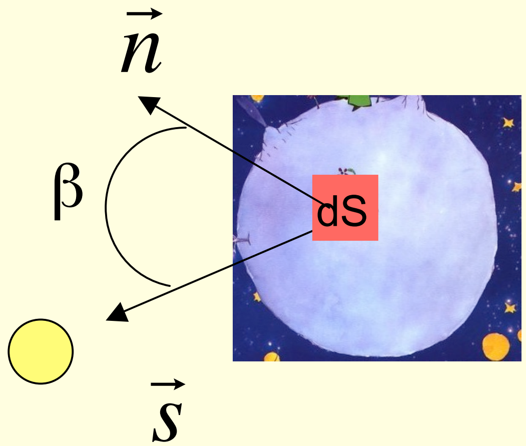
$$\vec{F} = -\frac{\Phi_S}{c}\iint_{S-Sun}\left[(1-\rho)\cos\beta\,\vec{s} + \left(\frac{2}{3}\delta + 2\rho\cos\beta\right)\cos\beta\,\vec{n}\right]dS$$

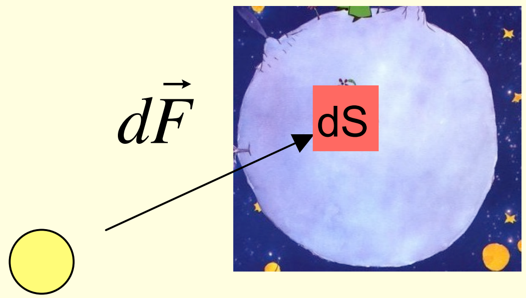
dS = element of the outer surface
$\vec{s}$ = direction of the Sun
$\vec{n}$ = normal to the surface

$$\vec{s}\bullet\vec{n} = \cos\beta$$

S-Sun = portion of the outer surface
Illuminated by the Sun

Absorption : α





$$d\vec{F} = -\frac{\Phi_S}{c}\alpha \cos\beta \, dS \, \vec{s}$$

Transfer of linear momentum from the photons to the surface due to absorption

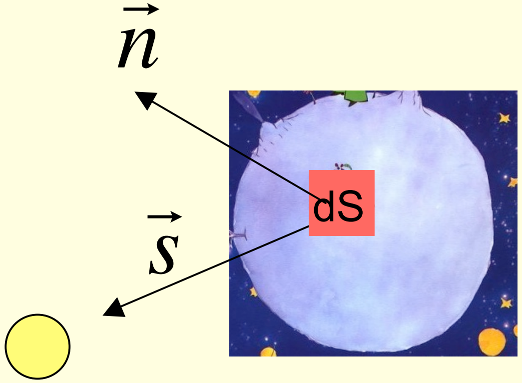
Cross section of the surface = cos β dS

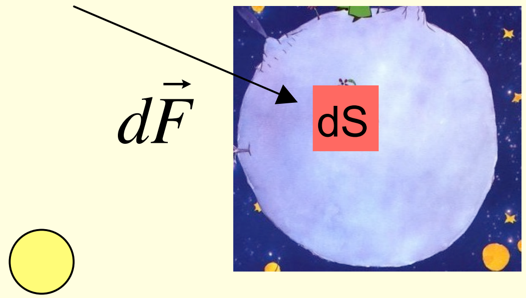
Asteroids : α > 1/2

Cos β > 0 required for the illumination (convex shape)

$\vec{s} \bullet \vec{n} = \cos\beta$

Reflection : ρ

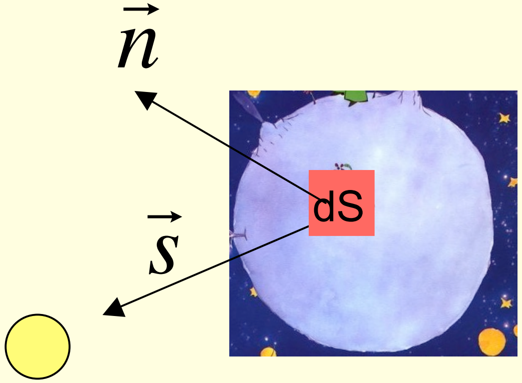


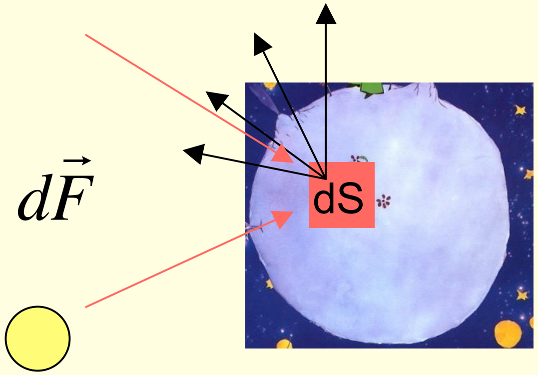


$$d\vec{F} = -\frac{\Phi_S}{c}\rho\, 2\cos^2\beta\; dS\; \vec{n}$$

Transfer of linear momentum + recoil momentum

Diffusion : δ





$$d\vec{F} = -\frac{\Phi_S}{c}\delta \ \cos\beta \ dS\,(\vec{s} + \frac{2}{3}\vec{n})$$

$$\frac{\int_0^{2\pi} d\lambda \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta}{\int_0^{2\pi} d\lambda \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta} = \frac{2}{3}$$

First step : absorption of the photons (direction of the Sun) $-\vec{s}$

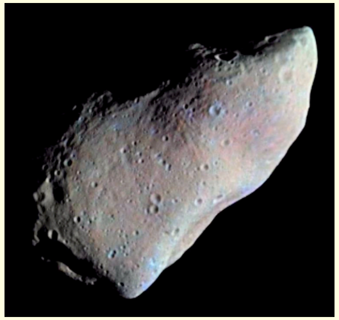
Second step : reemitted in different directions (resultant = direction of the normal) $-\vec{n}$

Transfer of linear momentum : 2 components

Total force : direct radiation pressure

$\alpha + \rho + \delta = 1$

$$\vec{F} = -\frac{\Phi_S}{c} \iint_{S-Sun} \left[(1-\rho)\cos\beta\, \vec{s} + (\frac{2}{3}\delta + 2\rho\cos\beta)\cos\beta\, \vec{n} \right] dS$$



Shape of the asteroid : no symmetry

→ $\vec{F}$ not applicable to CM

α, ρ and δ not necessary constants

Ideal case : sphere radius R with constant α, ρ and δ

$$\vec{F} = \frac{\Phi_S}{c} A\, \vec{r}$$

with $A = (\alpha + \rho + \frac{13}{9}\delta)\, \pi R^2 > \pi R^2$ effective cross section

change in the gravitational mass of the Sun
(radial force, intensity inversely proportional to the distance to the Sun)

→ Quasi-periodic terms in time - no secular effects

Remark : A / m coefficient

- Geostationnary debris
- 2 body problem + direct radiation pressure
- Numerical integrations by Anselmo (2005)
- Very large A/m

Sphere : $A = 4 \pi R^2$

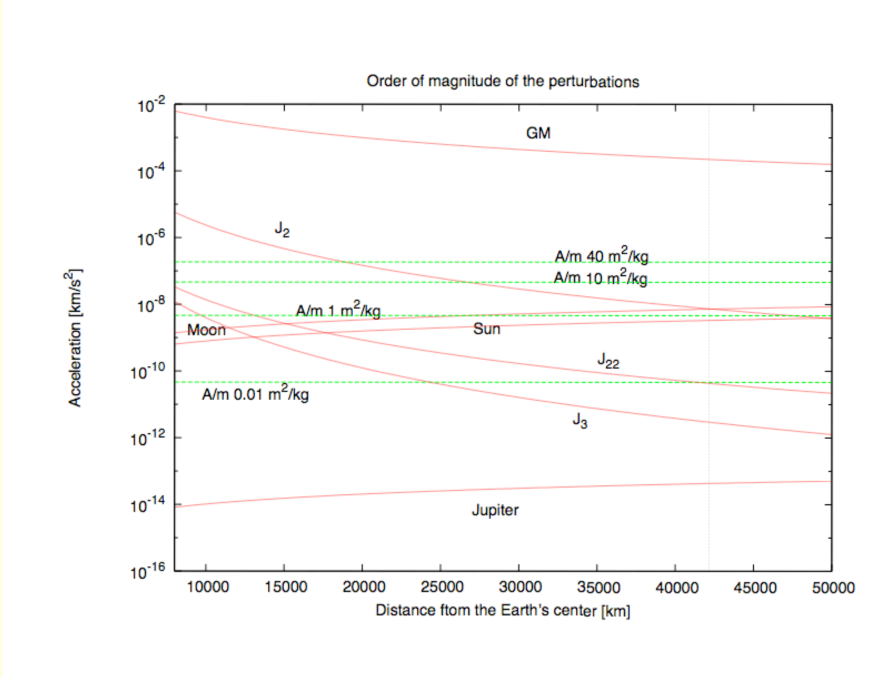
$m = 4/3 \pi \rho R^3$

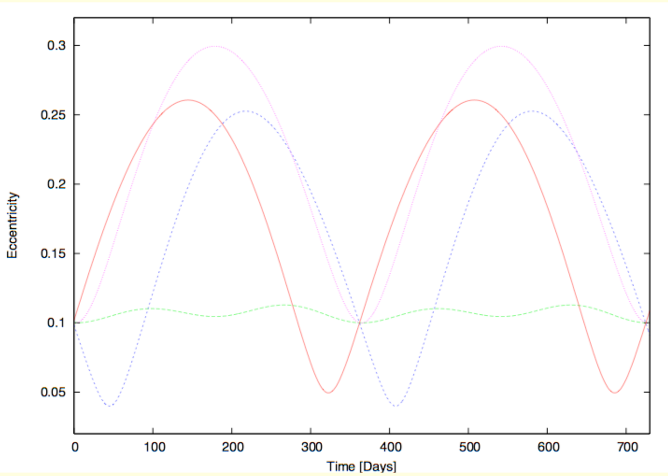
$A / m = 3 / \rho R$

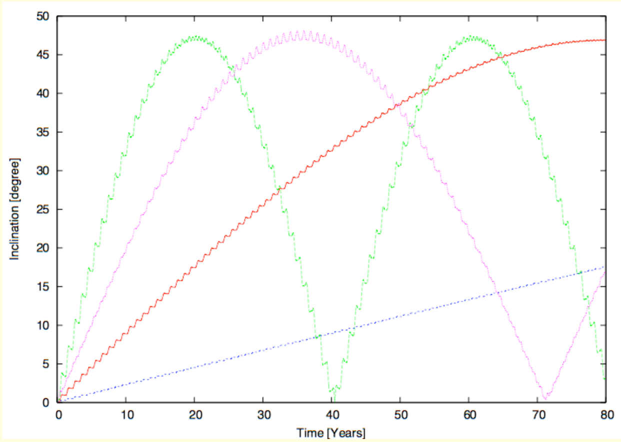
Square Thin Plate : $A = 2 R^2 + 2 R \varepsilon + 2 R \varepsilon$

$m = R^2 \varepsilon \rho$

$A / m = 2 / \rho \varepsilon$







Secular effects on the semi-major axis

2- body problem + non gravitational force :

$$\vec{a} = -\frac{GM}{r^3}\vec{r} + \frac{\vec{\mathrm{F}}}{\mathrm{m}}$$

(R,T,W)

Reference frame linked to the orbital plane

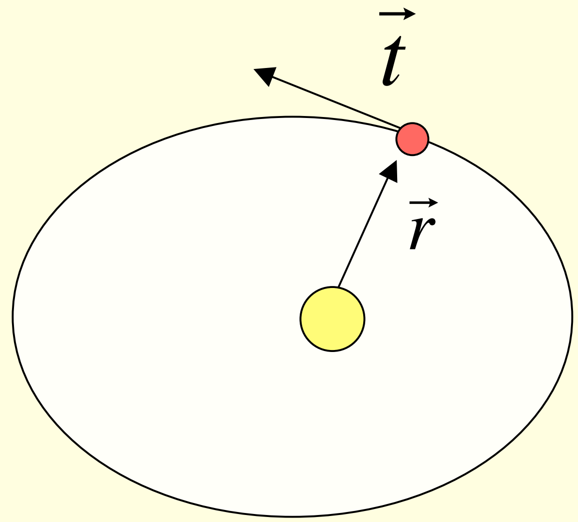
$$\left(\vec{u} = \frac{\vec{r}}{r},\ \vec{t} = \vec{w} \times \vec{u},\ \vec{w} = \frac{\vec{l}}{l} = \frac{\vec{r} \times \vec{v}}{l}\right)$$

$$\frac{dE}{dt} = \vec{F} \bullet \vec{V} = R\,V_R + T\,V_T + W\ 0$$

$$= \frac{GM}{l}\left(T + e\,(R \sin f + T \cos f)\right)$$

$$\frac{GM}{2a^2}\frac{da}{dt}$$

f = true anomaly
n = mean motion



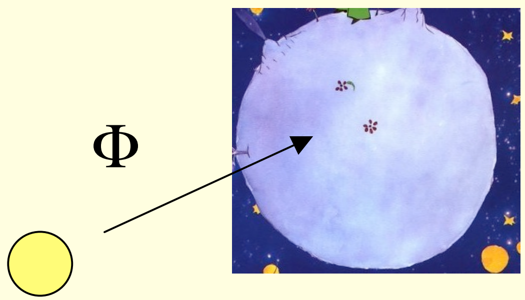
for a circular orbit

$$\Delta a = \frac{4\pi}{n^2}\overline{T}$$

= 0 if the body is symmetric - regular - ideal
= average contribution of T (along track)

$$Acc = a\frac{d^2M}{dt^2} = a\frac{dn}{dt} = -3T + o(e)$$

Thermal emission



Transformation of the absorbed fraction of $\alpha\Phi$ into heat (T)

Surface temperature not constant over the surface and with time

Reemission of thermal radiation

➡ linear momentum ➡ acceleration

Corresponding force for each dS of the surface

$$d\vec{F} = -\frac{2\,\varepsilon\,\sigma\,T^4}{3\,c}\,dS\,\vec{n}$$

σ = constant of Stephan-Boltzmann
ε = emissivity (=1 for black body)

Perfect sphere with isothermal surface : $\iint_S d\vec{F} = \vec{0}$

$\iint_S d\vec{F}$ < direct radiation pressure (In most cases)
= for dark objects (a = 1) C- type asteroids

Yarkovsky secular effect

- Thermal properties of the body
- Optical properties of the surface
- Illumination from the Sun
- Orbital dynamics
- Time Delay between maximum illumination and maximum temperature

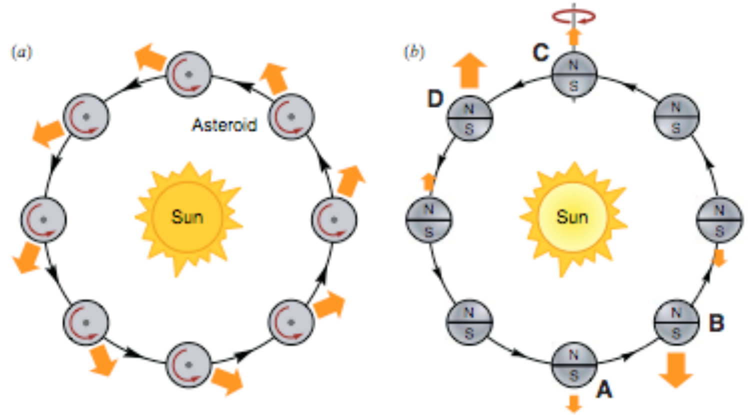


Figure 1

(a) The diurnal Yarkovsky effect, with the asteroid's spin axis perpendicular to the orbital plane. A fraction of the solar insolation is absorbed only to later be radiated away, yielding a net thermal force in the direction of the wide arrows. Because thermal reradiation in this example is concentrated at about 2 PM on the spinning asteroid, the radiation recoil force is always oriented at about 2 AM. Thus, the along-track component causes the object to spiral outward. Retrograde rotation would cause the orbit to spiral inward. (b) The seasonal Yarkovsky effect, with the asteroid's spin axis in the orbital plane. Seasonal heating and cooling of the "northern" and "southern" hemispheres give rise to a thermal force, which lies along the spin axis. The strength of the reradiation force varies along the orbit as a result of thermal inertia; even though the maximum sunlight on each hemisphere occurs as A and C, the maximum resultant radiative forces are applied to the body at B and D. The net effect over one revolution always causes the object to spiral inward.

- Spherical body
- Circular orbit
- da/dt >0 (da/dt <0)

Diurnal effect

- Spherical body
- Circular orbit
- da/dt <0

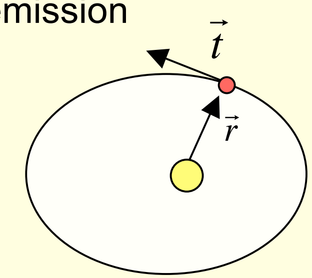
Seasonal effect

$$< \frac{da}{dt} >_{diurnal} = \frac{8}{9n} \alpha\, \Phi\, F_w \cos\gamma + O(e)$$

$$< \frac{da}{dt} >_{seasonal} = -\frac{4}{9n} \alpha\, \Phi\, F_n \sin^2\gamma + O(e)$$

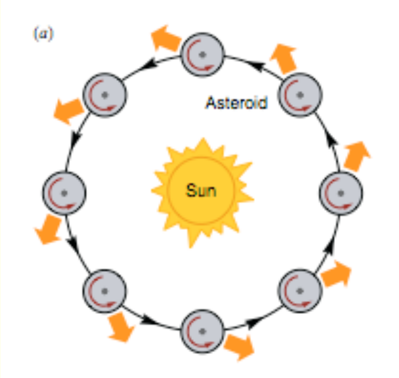
$$F_w \text{ and } F_n \geq 0$$

Averaged along track component of the thermal emission



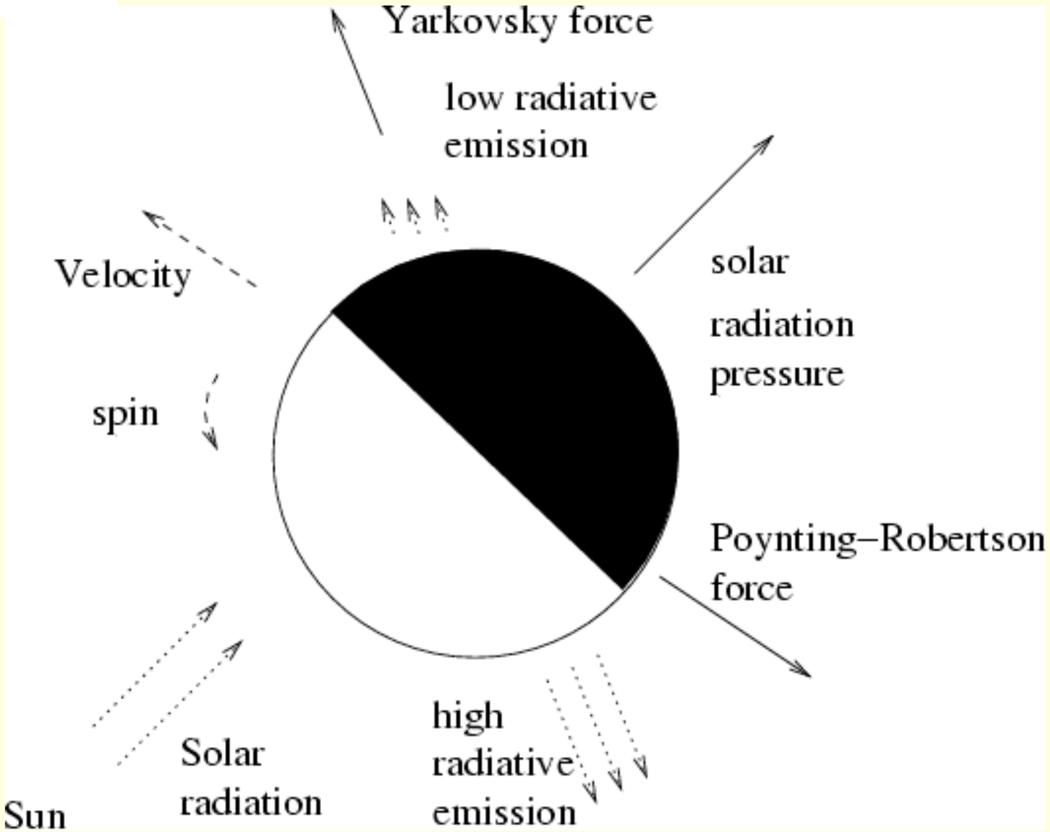
γ is the (orbital) obliquity of the spin axis
ω is the rotation frequency, n the revolution frequency

Diurnal Yarkovsky (prograde rotation)



$$\vec{F} = \frac{\Phi_S}{c} A \vec{r}$$

Direct Radiation Pressure



$$\vec{F}_{PR} = -\frac{\Phi}{c^2}\vec{V}$$

Poynting-Robertson drag

YORP effect

Yarkovsky - O'Keefe- Radzievskii - Paddack

$$d\vec{F} = -\frac{2\,\varepsilon\,\sigma\,T^4}{3\,c}\,dS\,\vec{n}$$

$$\vec{F} = \iint_S d\vec{F} \qquad \vec{T} = \iint_S \vec{r} \times d\vec{F}$$

Sunlight alters spin

- YORP = corresponding thermal Torque
- Modifications of spin rates and axis orientations (obliquities)
- Asteroids with irregular shapes : sphere + 2 wedges
- Kicks of the photons on the wedges are in different directions
- The asteroid spins up (or the opposite)
- Asteroid = surface made of N triangles

Farinella - Vokrouhlicky - Rubincam - Bottke - Nesvorny - Morbidelli

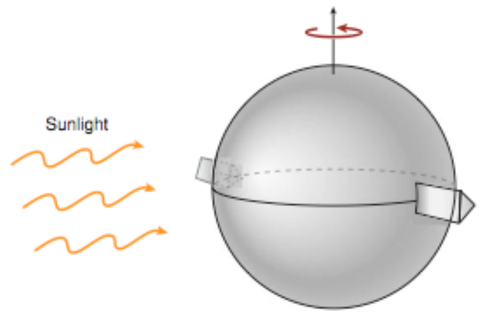


Figure 5

Spin up of an asymmetrical asteroid. The asteroid is modeled as a sphere with two wedges attached to its equator. The asteroid is considered a blackbody, so it absorbs all sunlight falling on it and then reemits the energy in the infrared as thermal radiation. Because the kicks produced by photons leaving the wedges are in different directions, a net torque is produced that causes the asteroid to spin up.

Comparison

Asteroid NEA
D = 500 m
ρ = 2 gr / cm^3
M = 200 10^6 kg

n = 0.012 /day

Max Radiation Force = 4 10 $^{-11}$ m / sec^2 at 1 AU

$$\Delta a = \gamma\ 3{,}3\ m\ in\ a\ day$$

$$= \gamma\ 100\ m\ in\ a\ month$$

$$\gamma = \cos\left(\sum \vec{F}_{non\ grav}, \vec{V}\right)$$

γ = asymmetry of the surface
= 0 for a “ideal” body
γ changes slowly with time

Measured Yarkovsky : $\Delta\ a = 100\ m\ in\ a\ year$

(γ is only a few percent : average T with respect to the max)

a = 3,156 AU
e = 0,156
i = 2,2°

Shape of the asteroids - binaries

ρ= 1.25 gr / cm^3 (30 % empty space)
r = 171 kms
R = 86 kms

Period = 16.5 days

90 Antiope Twin rubble pile ?
(IMCCE + Berkeley)

Elongated rubble pile (Themis catastrophic event - 2.5 10^6 years)
-- > two egg-shaped rubble piles (mutual gravitation)
-- > each a Roche ellipsoid (hydrostatic shape) (< 7% spheres)

P. Descamps (IMCCE) (March 2007)
The formation of such a large double system is an improbable event and represents a formidable challenge to theory

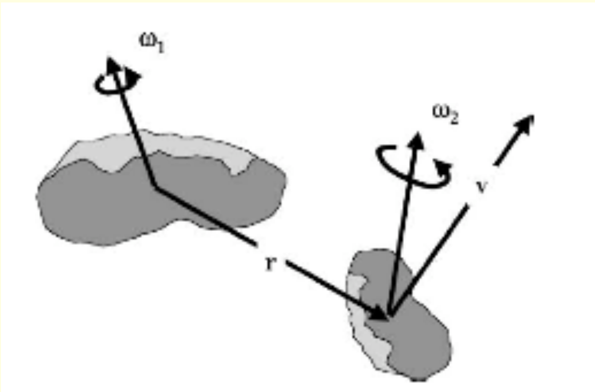


FIG. 1. Geometry of the full two-body problem.

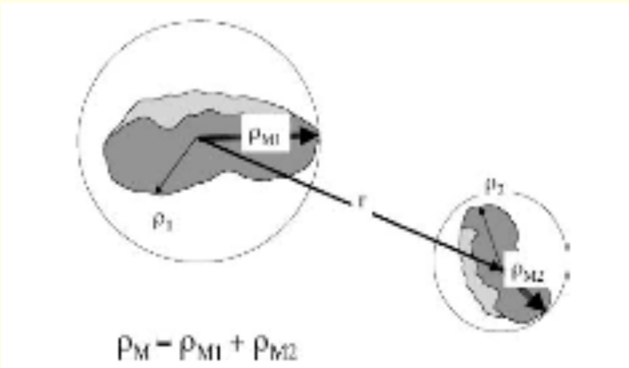


FIG. 2. Radius bounds on the bodies.

Triple asteroid (Silvia)

Scheeres 2002

size - shape - density - mass

Conclusions

- Ephemerides : individual
- Strongly dependent on non gravitional perturbations
- Radiation pressure and thermal emission
- Dependence on the properties of the surface, the conductivity, the temperature
- Dependence on the spin rate, orientation of spin axis and orbital parameters
- All these parameters are functions of time
- Coupled problem : orbit and rotation
- Dependent on the masses and densities

Real challenge for the scientific community

Tholen classification (refined by SMASS)

The most widely used taxonomy for over a decade has been that of David J. Tholen in 1984.

This classification was developed from broad band spectra (between 0.31µm and 1.06µm)
obtained during the Eight-Color Asteroid Survey (ECAS) in the 1980s,
in combination with albedo measurements.

The original formulation was based on 978 asteroids.

This scheme includes 14 types with the majority of asteroids falling into one of three broad categories

C-group dark carbonaceous objects, including several sub-types

B-type, F-type, G-type
C-type the remaining majority of 'standard' C-type asteroids.

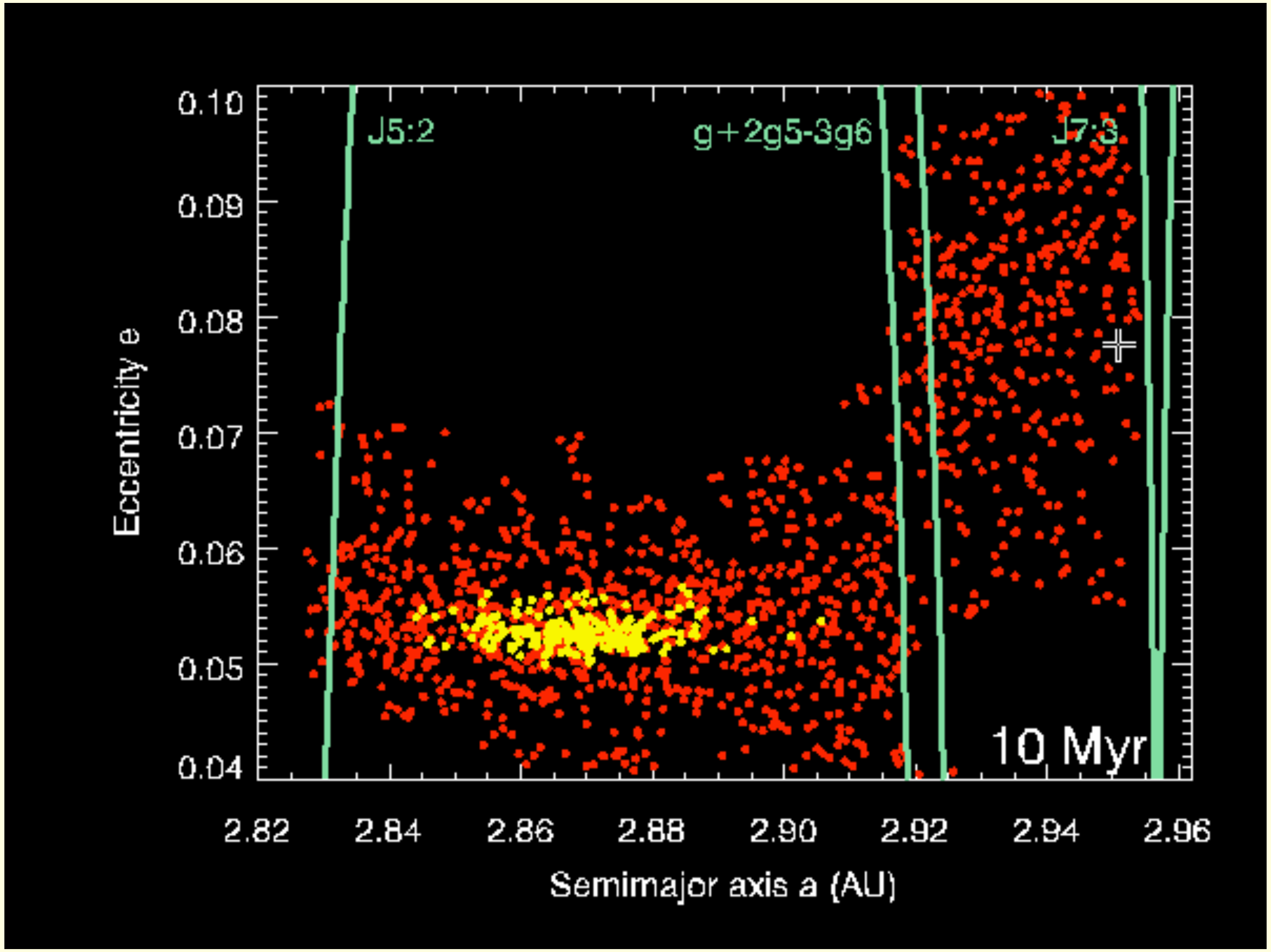
This group contains about 75% of asteroids in general.

S-type silicaceous (i.e. stony) objects.

This class contains about 17% of asteroids in general.

X-group

- M-type metallic objects, the third most populous group.
- E-type differ from M-type mostly by high albedo
- P-type differ from M-type mostly by low albedo



J5:2
g+2g5-3g6
J7:3
10 Myr
Eccentricity e
Semimajor axis a (AU)
0.10
0.09
0.08
0.07
0.06
0.05
0.04
2.82
2.84
2.86
2.88
2.90
2.92
2.94
2.96

(Bottke 2002)