Rtns Vigo, 23-26 January 2017

RECENT TRENDS IN NONLINEAR SCIENCE

ABSTRACTS OF THE POSTER SESSION

Recent Trends in Nonlinear Science.
January 23-26, 2017, Vigo, Spain.

Starting and stopping vortices of an airfoil

Roberto Agromayor

Norwegian University of Science and Technology (NTNU), Norway

Abstract

The transient flow around a NACA4612 airoil profile was analyzed and simulated at a Reynolds number $Re = 1000$ and angle of attack $\alpha = 16°$ paying especial attention to the starting and stopping vortices shed from the airfoil. A detailed review of the underliying physics of the generation of lift was presented with focus on the importance of viscosity as the essential factor for the generation of lift. The incompressible Navier-Stokes equations with constant density and viscosity in an inertial frame of reference were solved with OpenFOAM using a linear upwind finite volume method (FVM) for the space discretization and the implicit Euler method for the time integration. Both structured and unstructured meshes of different sizes and number of cells were analyzed and the solution was shown to be grid independent. Lift and drag coefficients were computed as a function of time and the results were discussed. The results were verified using the Kelvin circulation theorem as the line integral of the velocity and as the surface integral of vorticity over an inviscid contour. Three flow animations were prepared with the simulation results and compared with the historical flow visualizations from Prandtl and Tietjens.

This is a joint work with Jairo Rúa.

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An existence theorem for fractional hybrid differential inclusions

Tahereh Bashiri

Universidade de Santiago de Compostela, Spain

Abstract

Fractional differential equations and inclusions have been of great interest recently. This is due to both the intensive development of the theory of fractional calculus itself and the applications of such constructions in various scientific fields such as physics, mechanics, chemistry, and engineering. The study of fractional differential inclusions was initiated by El-Sayed and Ibrahim. In the last few decades, the hybrid differential inclusions caught great attention. Usable instrument to develop the existence theory for the hybrid inclusions is multi-valued forms of hybrid fixed point theorems.

In this manuscript we investigate the existence of solution of to fractional hybrid differential inclusion(FHDI) by using a Dhage fixed point theorem. Also, an example is analyzed to show the use of the reported results.

This is a joint work with S. Mansour Vaezpour and Juan J. Nieto.

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Delay presence in natural phenomena

Sebastián Buedo-Fernández

Universidade de Santiago de Compostela, Spain

Abstract

It is well known that in many natural processes, such as population growth, the evolution of the system depends on past states of it, for example, with birth or development periods. When this delay effect is included, the dynamics of the system can change, even in the most simple cases, as can be shown in the references [1, 2, 3]. So, the modelling of these phenomena with differential equations should include the delay effect for realistic reasons. This study also involves the knowledge of the properties of the zeros of some transcendental equations because of the linearization method.

It will be proposed and analysed a simple model of population growth divided by age groups in an environment where a predator is present, in order to show the difference between the ordinary and the delayed point of view.

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An easy procedure to solve analytically some fractional integral and differential equations

Daniel Cao-Labora

Universidade de Santiago de Compostela, Spain

Abstract

The first idea of a fractional derivative appears in a letter from l'Hpital to Leibniz in 1695, when the usual calculus was still being forged. However, the real interest and applications of the theory began at the 19^{th} century with the works of Abel. Probably, the lack of physical interpretations of fractional integrals and derivatives propitiated their oblivion during many decades. Nevertheless, there are some examples where the use of fractional calculus has perfect sense. For instance, a fractional integral equation appears in the work of Abel in mechanics (see tautochrone problem in [1] or, in a more informal way, in [2]) and a fractional differential equation appears when applying integral transforms and substitutions in some PDEs (see Bagley-Torvik equation in [3]).

These equations that were previously mentioned are examples of the fractional analogue to the well-known linear OIEs/ODEs with constant coefficients. Surprisingly, its correct resolution is not trivial and there are harsh methods to find the solutions (see [1]). Under some hypotheses on the elements of the equation, an alternative method is proposed to compute the solutions.

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A theory of linear differential systems with reflection

F. Adrián Fernández-Tojo

Universidade de Santiago de Compostela, Spain

Abstract

We develop a theory of linear differential systems analogous to the classical one for ODEs, including the obtaining of fundamental matrices, the development of a variation of parameters formula and the expression of the Greens functions. We also derive interesting results in the case of differential equations with reflection and generalize the Hyperbolic Phasor Addition Formula to the case of matrices. Finally, we prove a Liouville's formula for order two systems with reflection and conjecture a similar expression for the order n case.

This poster is based on a submitted paper with Professor A. Cabada and on-going work with Professor S. Codesido.

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Lower bound for the local cyclicity of quintic planar polynomial vector fields

Luiz F. S. Gouveia

Universitat Autònoma de Barcelona, Spain

Abstract

Hilbert early last century presented a list of problems that almost all of them are solved. One problem that remains open is the second part of the 16th Hilbert's problem: It is to determine the maximal number (named $H(n)$) of limit cycles, and their relative positions, of a planar polynomial systems of degree n

$$\left\{\begin{array}{rcl}\dot{x} & = & -y + P_n(x,y),\\ \dot{y} & = & x + Q_n(x,y).\end{array}\right.$$

We are interested here in the local version of this Hilbert's 16th problem that consist in to provide the number $M(n)$ of small amplitude limit cycles bifurcating from an elementary center or an elementary focus. Clearly $M(n) \leq H(n)$. See more details in [6].

For $n = 2$, Bautin proved in [4] that $M(n) = 3$. Sibirskii in [5] proved that for cubic systems without quadratic terms there are no more than five limit cycles bifurcating from one critical point. In [6, 7] Zoladek found an example where eleven limit cycles could be bifurcated from a single critical point of a cubic system and Christopher, with the technique presented in [1], gave a simpler proof of Zoladek's result perturbing a Darboux cubic center.

We prove that $M(5) \geq 33$. In particular, we present a center such that 33 limit cycles bifurcate from the origin. We remark that this lower bound coincides with the value, $M(n) = n^2 + 3n - 7$, conjectured by Giné in [2]. The computations have been done using a generalizaion of the parallelization procedure, introduced by Liang and Torregrosa in [3], for finding the higher order terms in the perturbation parameters.

This is a joint work with Joan Torregrosa.

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On unbounded solutions of singular IVPs with ϕ-Laplacian

Lucía López-Somoza

Universidade de Santiago de Compostela, Spain

Abstract

We study analytical properties of a singular nonlinear ordinary differential equation with a ϕ-Laplacian. Bounded solutions for this problem have already been studied so our main goal will be to study unbounded solutions and provide conditions for their existence. We will consider two different cases: uniqueness and lack of uniqueness of solution.

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Degree theory for discontinuous operators

Jorge Rodríguez-López

Universidade de Santiago de Compostela, Spain

Abstract

We introduce a new definition of topological degree for a meaningful class of operators which need not be continuous. Subsequently, we derive a number of fixed point theorems for such operators.

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The role of differential inclusions in the study of uncertain differential equations

Rosana Rodríguez-López

Universidade de Santiago de Compostela, Spain

Abstract

In many situations, when we try to obtain a mathematical model to predict the evolution of a real process, an important difficulty we face is the influence of imprecise data and factors. One possibility to handle this uncertainty through a mathematical model is the use of fuzzy differential equations. In the last years, the interest in this topic has increased considerably, and different concepts of derivatives for fuzzy-valued functions have been provided, starting with the notion of Hukuhara differentiability, and introducing other generalizations. We mention specifically strongly generalized differentiability and generalized Hukuhara differentiability [1, 2, 3, 4], due to the intensive study of the properties of solutions based on these notions (see, for instance, [3, 5, 7, 8, 9]).

A detailed analysis of the behavior of the solutions to differential models using different concepts of derivatives illustrates the essential differences of fuzzy models in contrast to their classical counterparts, even in the linear case, where we can easily detect that some models which are equivalent in the real case might be nonequivalent if we consider the corresponding fuzzy models [3].

Other approaches to interpret fuzzy differential equations do not require the definition of a derivative for fuzzy-valued functions, that is the case of the techniques based on Zadeh's Extension Principle or differential inclusions [6]. Recently, many authors have been investigating the relations between the different approaches available, in order to establish connections between the expressions of solutions to a same problem under several procedures.

We focus our attention on the expression of the explicit solution to some linear problems via the method of differential inclusions, illustrating the usefulness of its application and its strong connections with the solutions obtained following other different approaches.

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Characterization of Green's function constant sign for simply supported beam boundary conditions.

Lorena Saavedra

Universidade de Santiago de Compostela, Spain

Abstract

We consider the following linear differential problem

$$u^{(4)}(t) + p_1(t)\, u'''(t) + p_2(t)\, u''(t) + M\, u(t) = \sigma(t)\,, \quad t \in I \equiv [a,b]\,,$$
$$u(a) = u''(a) = u(b) = u''(b) = 0\,.$$

It is well-known the importance of ensuring that the related Green's function is of constant sign to obtain existence or stability results for the solutions, see [1].

The particular case where $p_1 \equiv p_2 \equiv 0$ on I has been studied in [4] for the negative case and in [2] for the positive. However, in both cases, the explicit expression of the Green's function was needed. Such an expression can be inapproachable in some cases.

Here, we establish a characterization for the Green's function constant sign, under suitable hypotheses, by means of spectral theory. The main achievement of our characterization is that the Green's function expression is not used.

References

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Arnold diffusion for several examples of perturbation using Scattering maps

Rodrigo G. Schaefer

Universitat Politècnica de Catalunya, Spain

Abstract

In this work we illustrate the Arnold diffusion for several examples of the *a priori* unstable Hamiltonian system of $2+1/2$ degrees of freedom

$$H(p,q,I,\varphi,s)=\frac{p^2}{2}+\cos q-1+\frac{I^2}{2}+h(q,\varphi,s;\varepsilon).$$

We prove that for any small periodic perturbation of the form $h(q,\varphi,s;\varepsilon)$, where

$$h(q,\varphi;\varepsilon)=\varepsilon\cos q\,(a_{00}+a_{10}\cos\varphi+a_{01}\cos s)$$

or

$$h(q,\varphi;\varepsilon)=\varepsilon\cos q\,(a_{00}+a_{10}\cos\varphi+a_{01}\cos(\varphi-s))$$

($a_{10}a_{01}\neq 0$ and $\varepsilon\neq 0$ small enough), there is global instability for the action, i.e., $I(0)\leqslant -I(\varepsilon)<I(\varepsilon)\leqslant I(T)$ for some T and for any positive $I(\varepsilon)\leqslant C\log\frac{1}{\varepsilon}$ for some constant C. For this, we apply a geometrical mechanism based in the so-called Scattering map.

We present some similarities and differences between these cases. Besides, we present a case with $3+1/2$ degrees of freedom, represented by the Hamiltonian

$$H(p,q,I_1,I_2,\varphi_1,\varphi_2,s)=\frac{p^2}{2}+\cos q-1+\frac{I_1^2}{2}+\frac{I_1^2}{2}+\varepsilon\cos q\,(a_{00}+a_1\cos\varphi_1+a_2\cos\varphi_2+a_3\cos s)\,.$$