An overview of Celestial Mechanics: from the stability of the planets to flight dynamics

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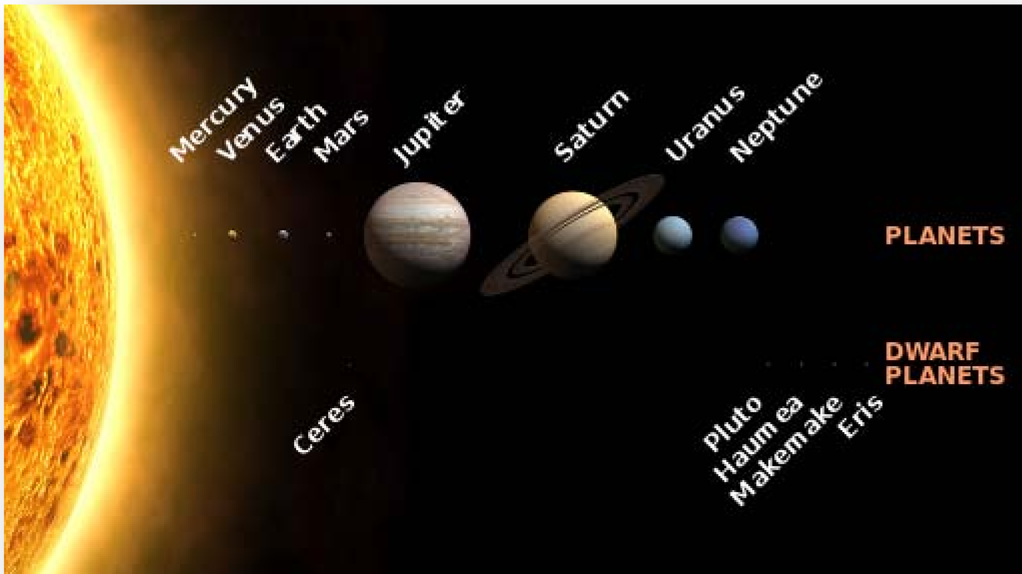
Image: CICLOPS, JPL, ESA, NASA

SUMMARY

- The Solar System
- Celestial Mechanics
- The 2-body problem
- The 3-body problem
- Chaos
- Orbital resonances
- Interplanetary highways
- Spin-orbit resonances
- Is the Solar system stable?

Image: S. Beckwith & the HUDF Working Group (STScI), HST, ESA, NASA

1. The solar system



- Sun
- Rocky planets
- Gas planets
- Dwarf planets
- Satellites
- Asteroids and comets

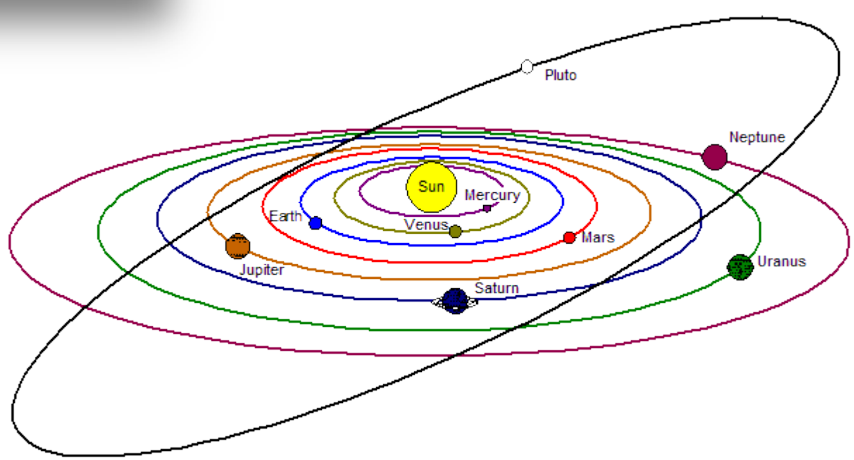
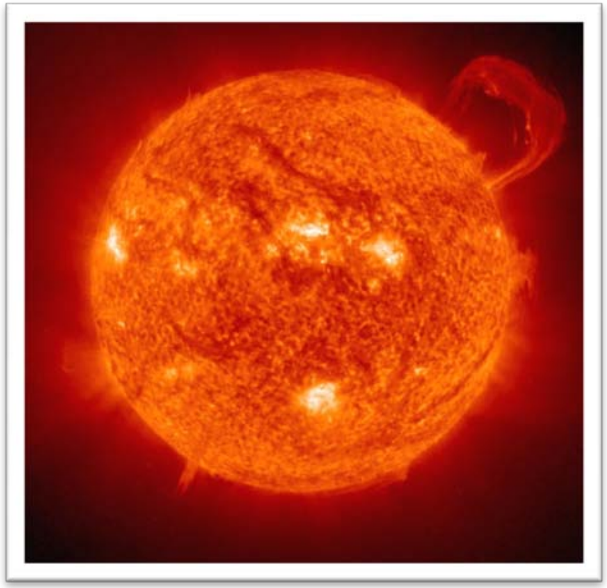


Image: NASA

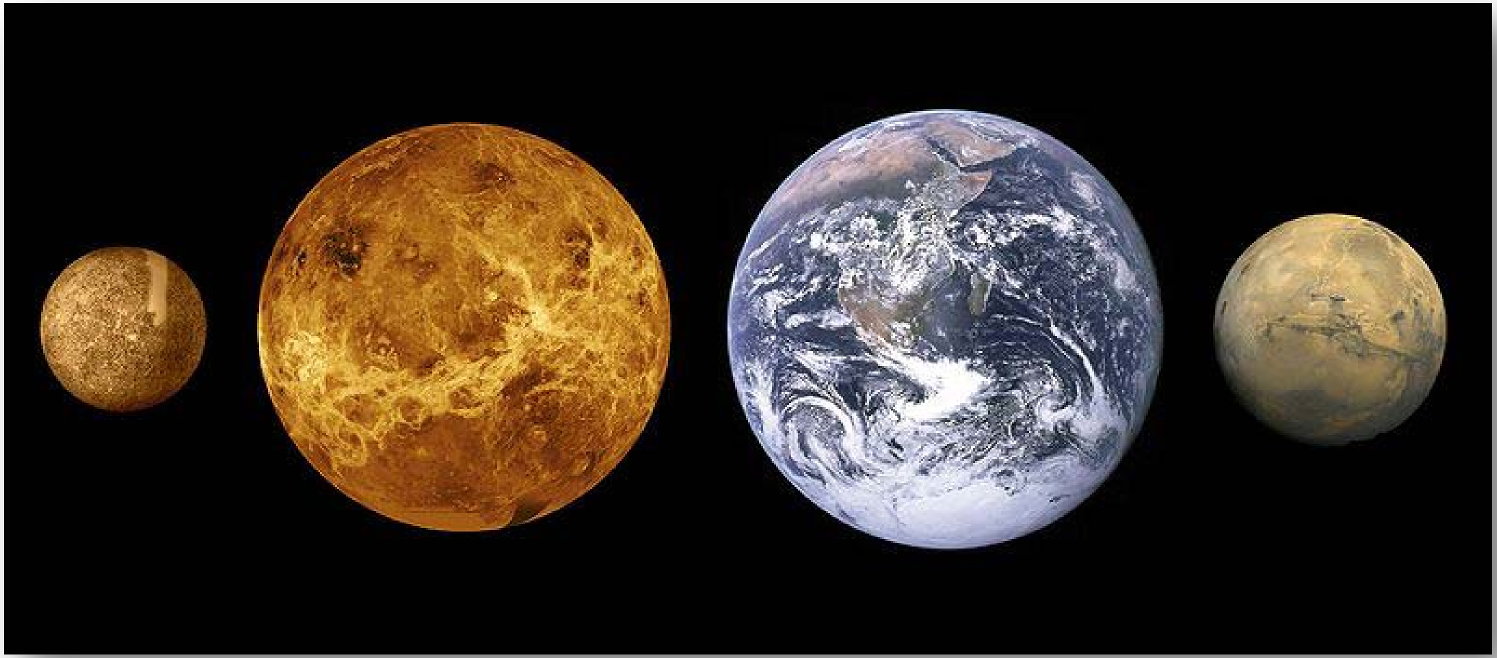
Il Sole



- Average size star at the border of a spiral arm of the Milky Way
- At about half of its life (4.5 billion years)
- Will evolve as a red giant and then a white dwarf
- Mass: $2*10^{30}$ Kg
- Radius 695,000 Km
- Composition: H - He

Images: ESA/NASA/SOHO, Hubble Heritage Team (STScI/AURA)

Internal rocky planets

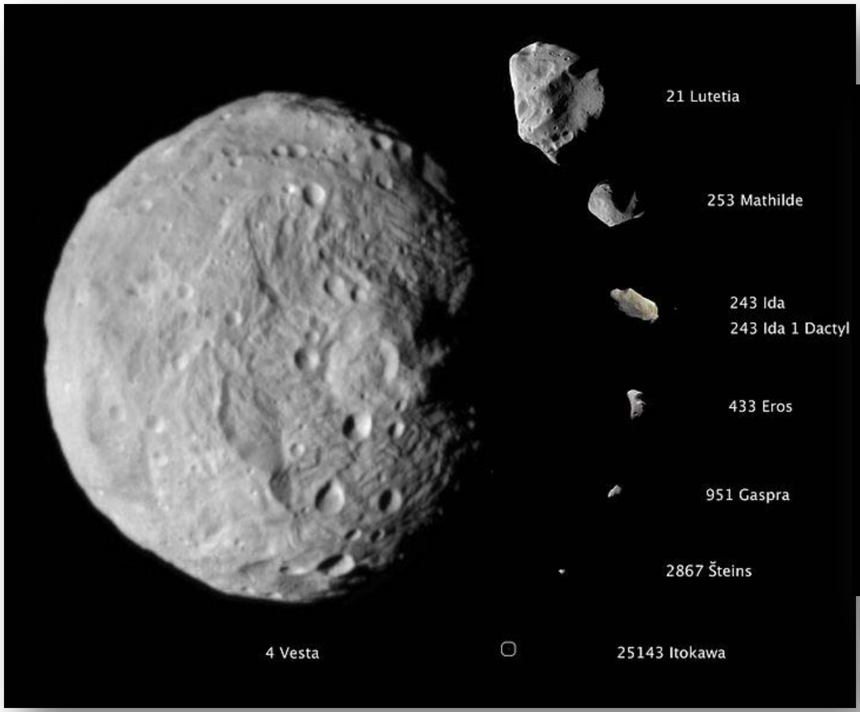


Mercury, Venus, Earth, Mars

Small, rocky, none or few satellites

Image: NASA

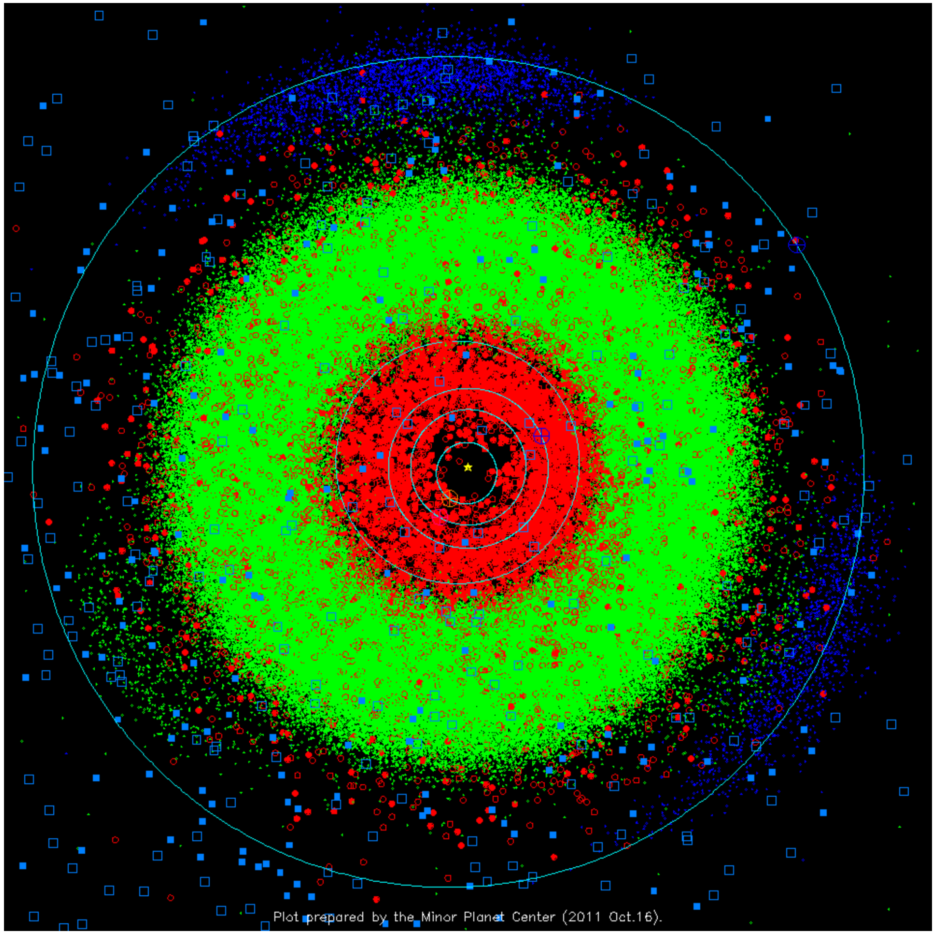
Asteroids: 580.000 oggetti catalogati



Between Mars and Jupiter, irregular forms and sizes, sometimes with satellites

Images: NASA, NEAR Project, Galileo Project

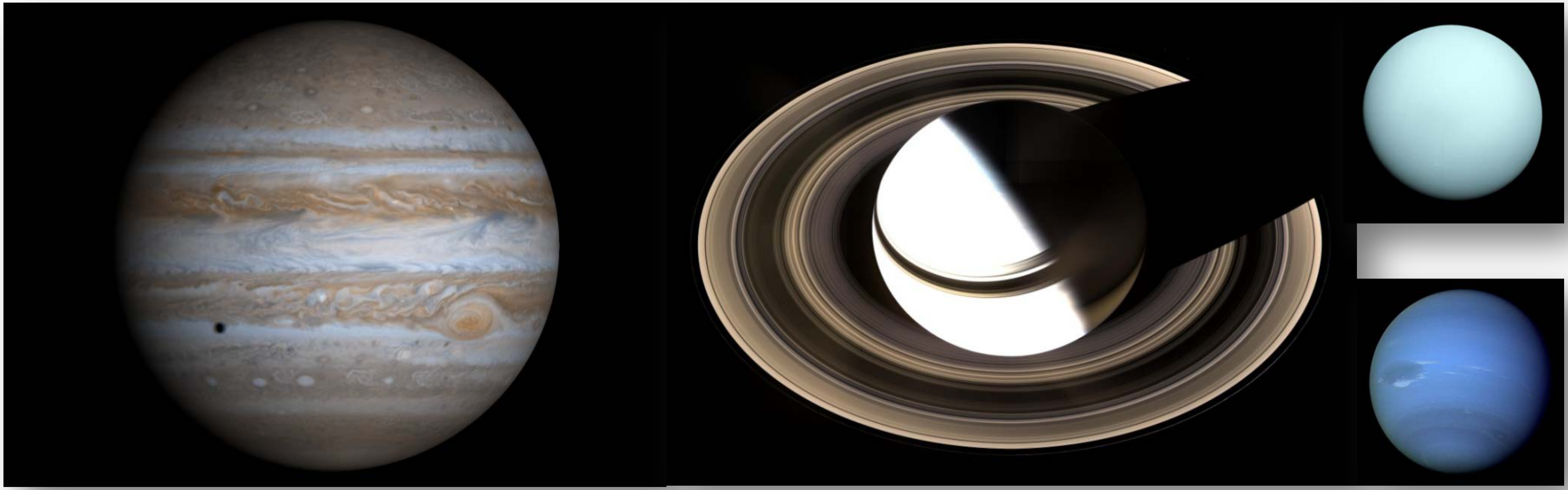
Internal Solar system



Asteroids-green, NEO-red, comets-blue.

Image: Minor Planet Center

External gaseous planets

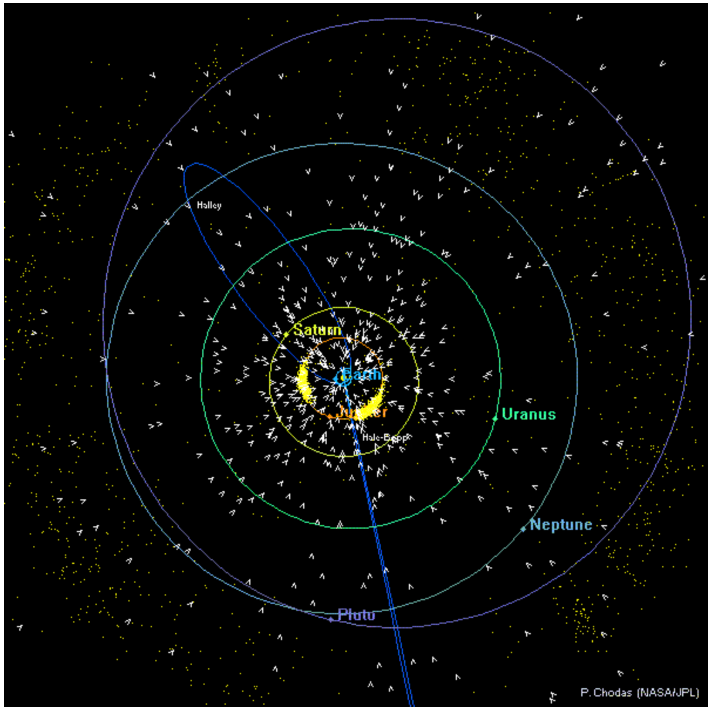


Jupiter, Saturn, Uranus, Neptune

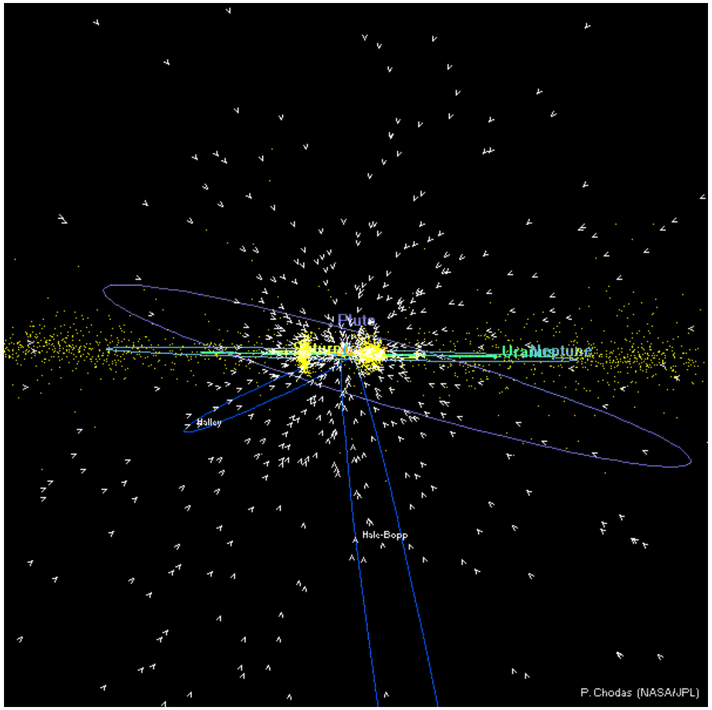
Big, gas, many satellites and rings

Images: NASA/JPL/University of Arizona, NASA/JPL/Space Science Institute, PD-USGOV-NASA, NASA

External Solar system: asteroids-yellow, in comets-white



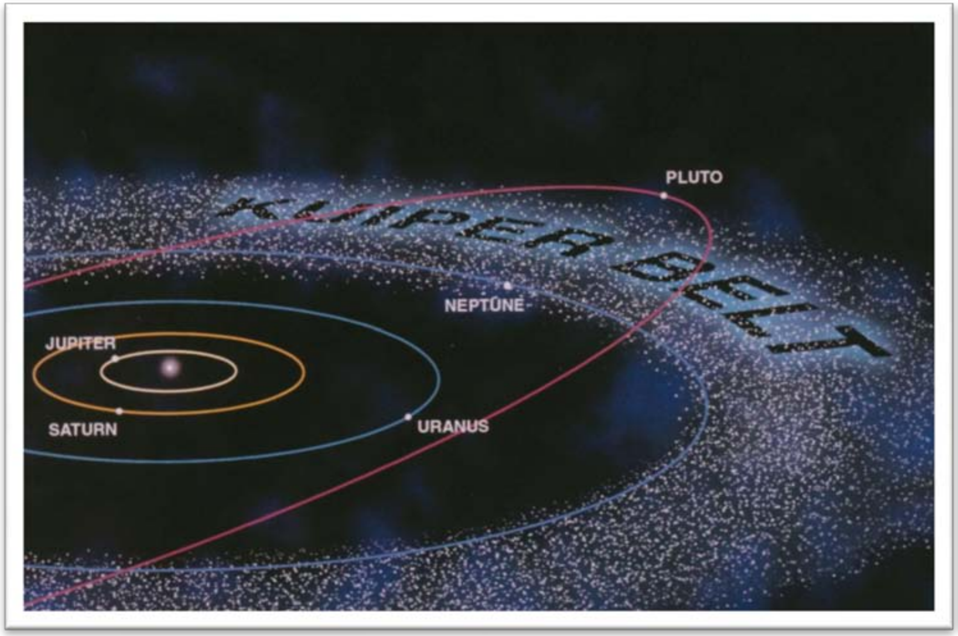
From above

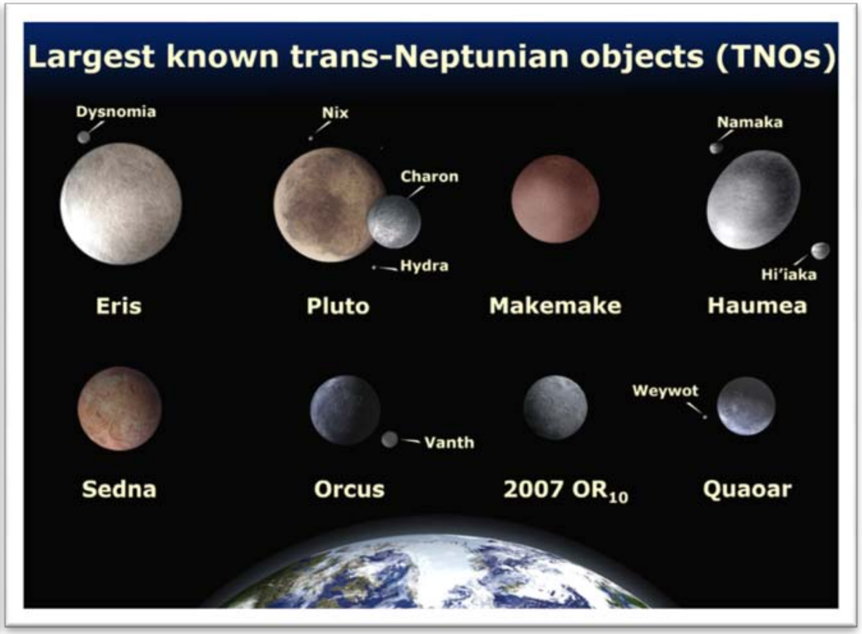


Profile

Images: P. Chodas, NASA/JPL

Kuiper belt

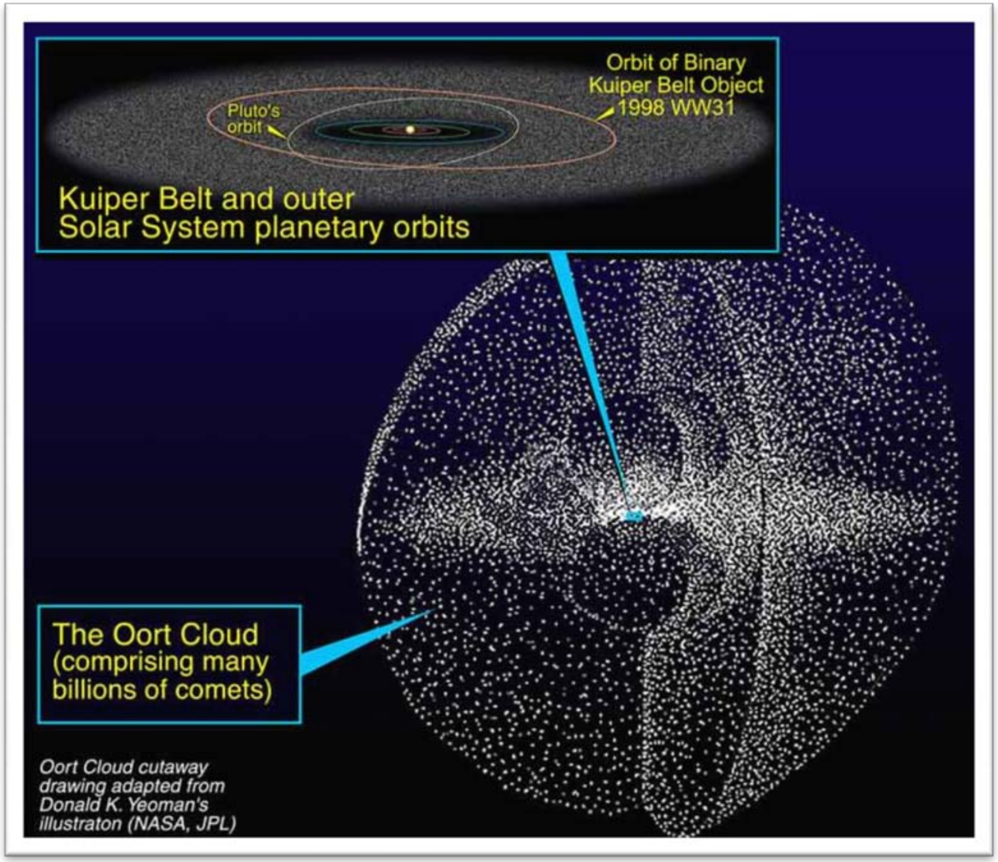




Thousands rocky-icy objects (among which Pluto), at the border of the Solar system

Images: NASA

Oort cloud



- 30,000 - 100,000 AU
- Billion icy objects
- Long-period comets, inserted in the Solar system by strong perturbations (close encounter of a star or passage of the Sun through a giant molecular cloud).

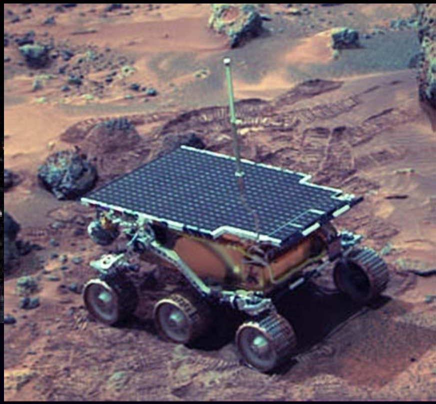
1 AU = Sun-Earth distance = 150 milion km.

2. Celestial Mechanics

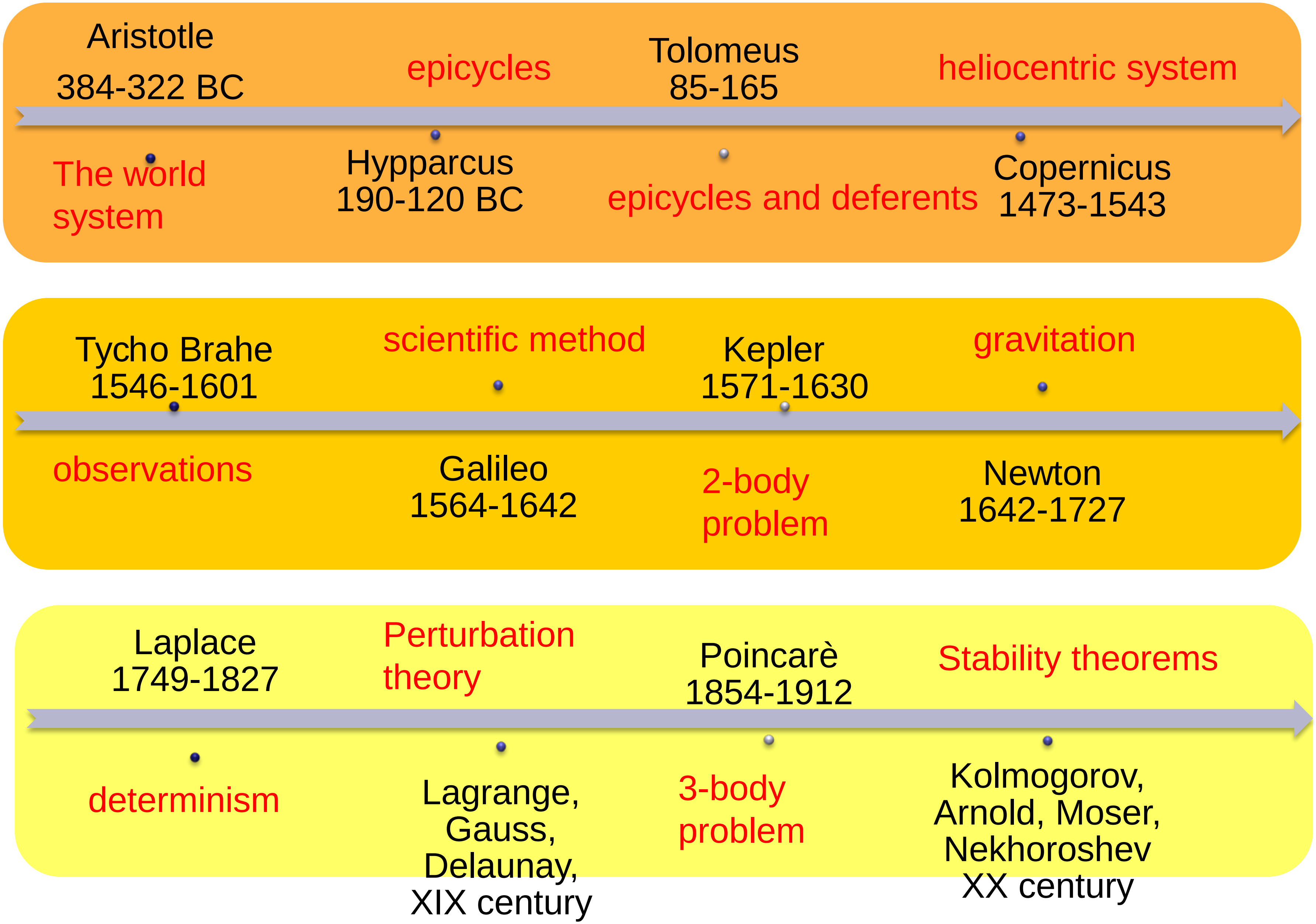
- **CELESTIAL MECHANICS** studies the dynamics of the objects in the Solar system: planets, satellites, asteroids, etc.

- **CELESTIAL MECHANICS** studies also the dynamics of **extrasolar planetary systems**

- **FLIGHT DYNAMICS** studies the motion of artificial satellites and interplanetary highways (first **space mission**: Sputnik 1 on 4 October 1957)



Immagini: NASA/JPL-Caltech/University of Maryland/Cornell, NASA and The Hubble Heritage Team (STScI/AURA), ESA-Hubble Collaboration, E. L. Wright (UCLA), The COBE Project, DIRBE, NASA , GALILEO/NASA/JPL


Aristotle
384-322 BC
The world system
epicycles
Hypparcus
190-120 BC
Tolomeus
85-165
epicycles and deferents
heliocentric system
Copernicus
1473-1543
Tycho Brahe
1546-1601
observations
scientific method
Galileo
1564-1642
Kepler
1571-1630
2-body problem
gravitation
Newton
1642-1727
Laplace
1749-1827
determinism
Perturbation theory
Lagrange, Gauss, Delaunay, XIX century
Poincarè
1854-1912
3-body problem
Stability theorems
Kolmogorov, Arnold, Moser, Nekhoroshev
XX century

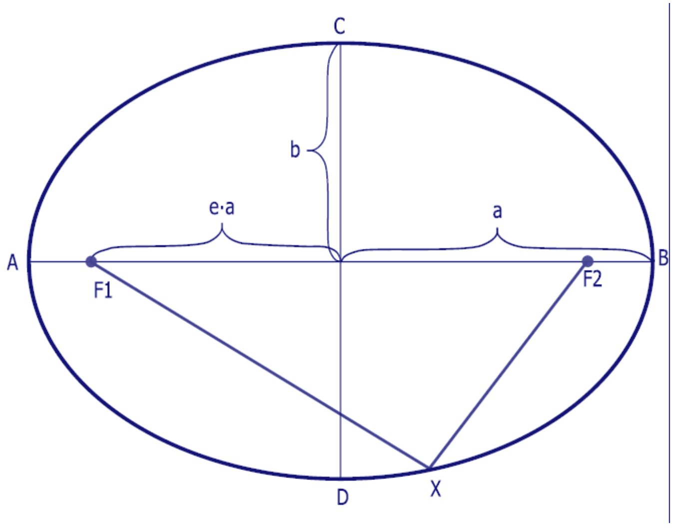
3. 2 body problem

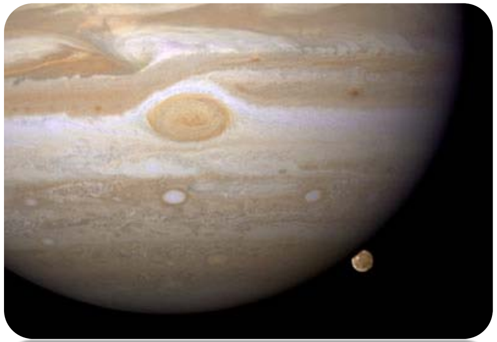
- Simplified model considering only the interaction between **2** objects

- Newton's law:

$$F = - \frac{G\,M\,m}{d^2}$$

- Kepler's laws





Immagini: NASA, NASA/NEAR

Johannes Kepler (1571-1630)

- Kepler believed in the heliocentric theory of Copernicus

- He wrote several books where astronomy was mixed with mathematics, physics, philosophy and music

- Studied for several years the astronomical data on the motion of the planets, collected byTycho Brahe (1546-1601), who built an astronomical observatory called "Uraniborg" – "The castle of the sky"

- Found 3 fundamental laws governing the 2-body problem

I Kepler law:

all planets move on ellipses with the Sun in one focus

kepler1.avi

Video: http://heasarc.nasa.gov/docs/heasarc/videos/education.html

II Kepler law:
all planets sweep equal areas in equal times

kepler2a.avi

kepler2b.avi

Video: http://heasarc.nasa.gov/docs/heasarc/videos/education.html

III Kepler law:

the square of the period of revolution is proportional to the cube of the semimajor axis

kepler3.avi

Video: http://heasarc.nasa.gov/docs/heasarc/videos/education.html

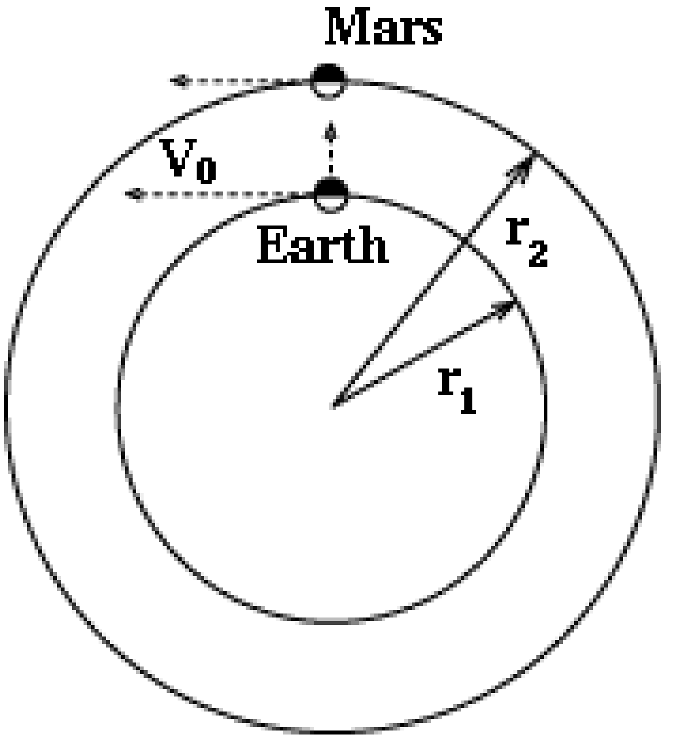
4. How NOT to go on Mars

- Earth and Mars on circular orbits with radii r_1, r_2
- Wait for Earth-Mars coonjunction and go on a straught line!

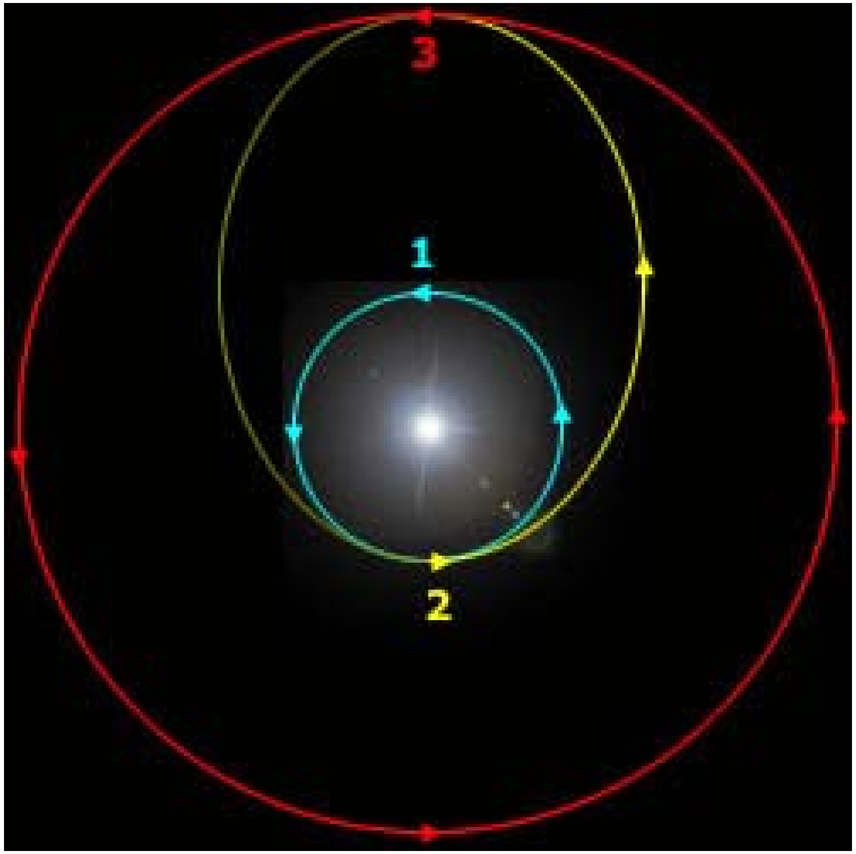
➢ Gravity curves the trajectories

➢ the orbit of Mars is reached perpendicolarly

➢ the Sun has a gravitazional influence on the satellite.

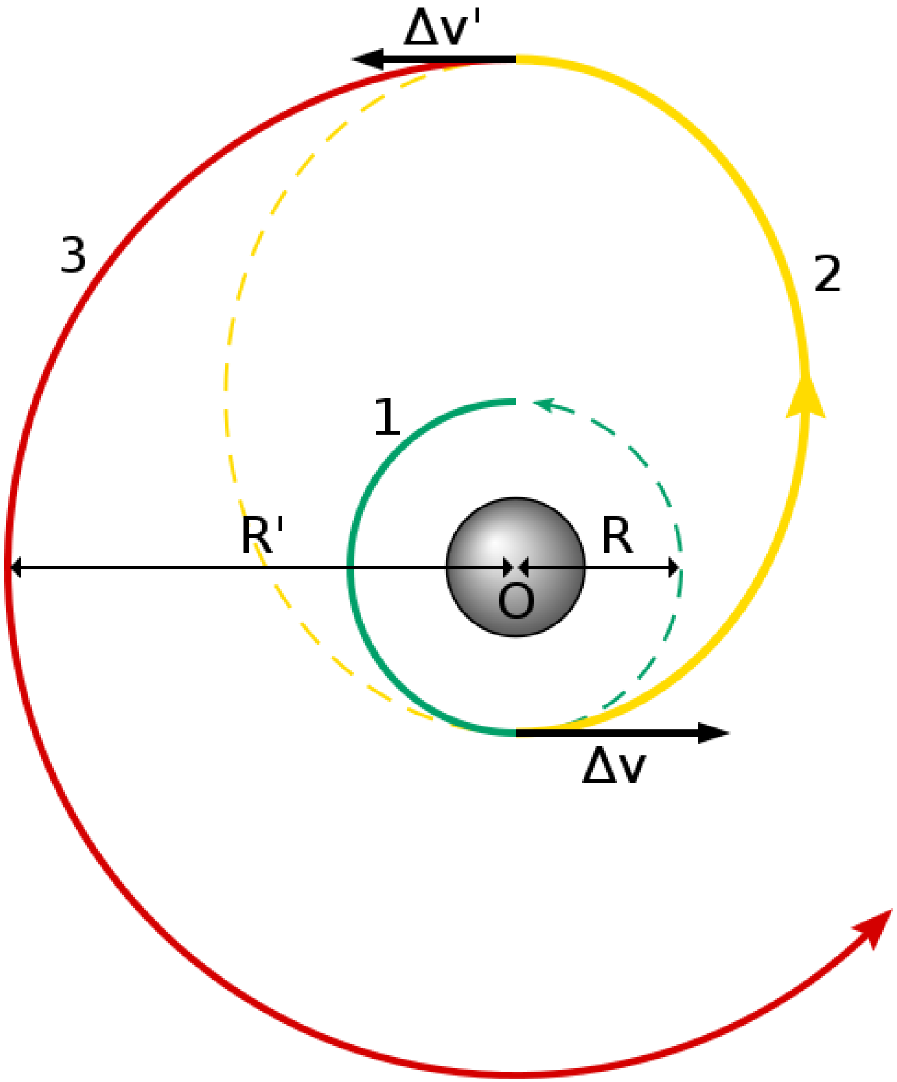


5. How to go on Mars



- Walter Hohmann (1880-1945) orbits
- 1 = initial orbit
- 2 = Hohmann transfer orbit
- 3 = target orbit

- Orbit 2 has perihelion on orbit 1 and aphelion on orbit 3
- Transfer with less fuel



- Switch the engines to insert the satellite in orbit 2 and then in orbit 3 (Δv)
- Δv measures the fuel consumption = cost of the missione
- L*aunch window* is the time interval to have that the satellite reaches Mars

6. The three body problem

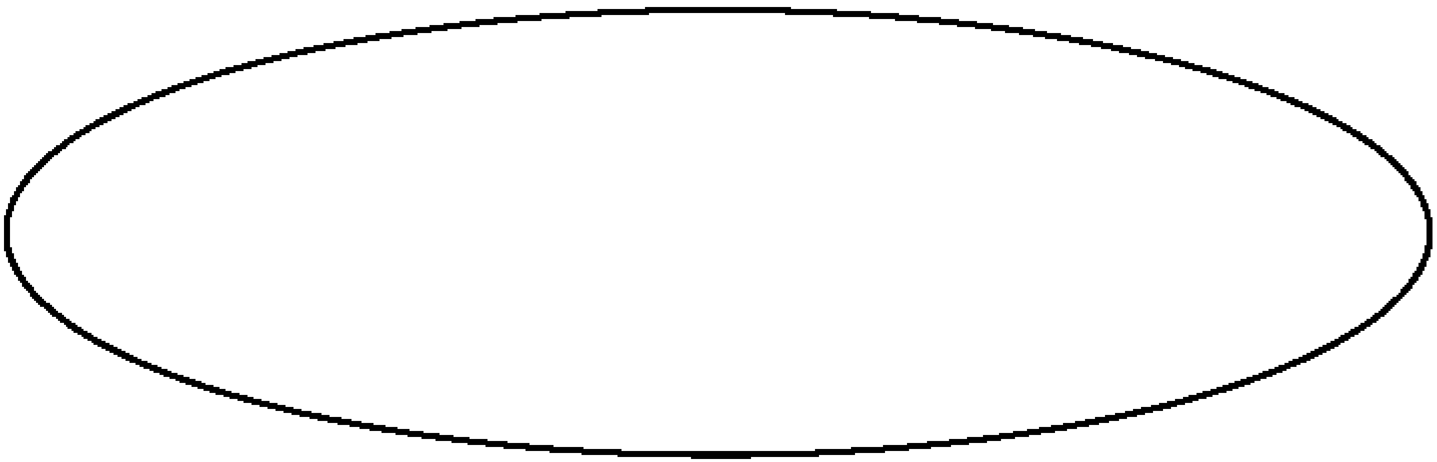
- What happens when we consider 3 bodies, e.g. Sun-Earth-Jupiter ?
- Kepler laws are only an approximation and the 3 body problem cannot be solved exactly!
- Perturbation theory: allows to compute *successive approximations* of the solution of the three body problem

- Sun-Earth-Jupiter : mass(*Jupiter)* = mass(*Sun)* / 1000 ➔

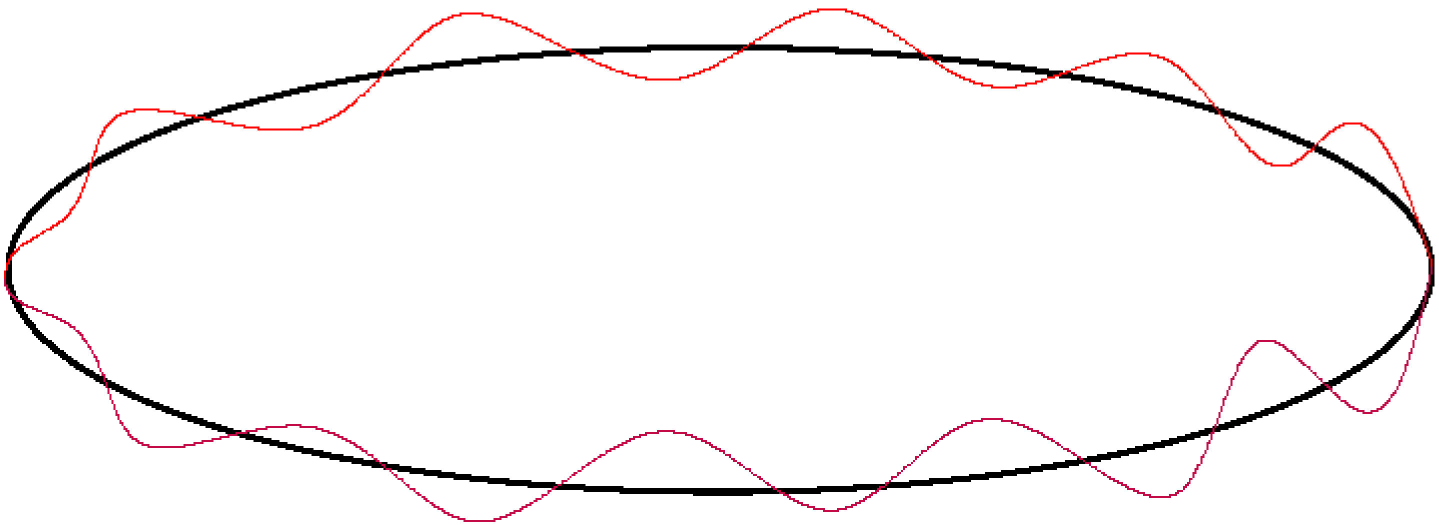
2-body Sun-Earth

+

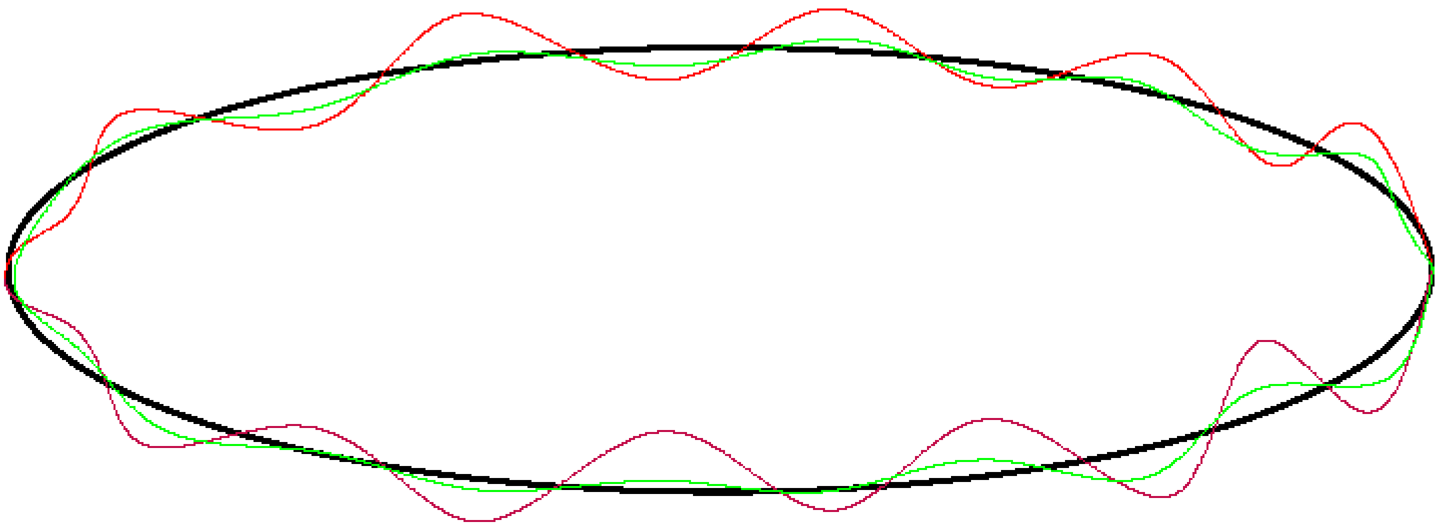
Small **perturbation due to Jupiter**



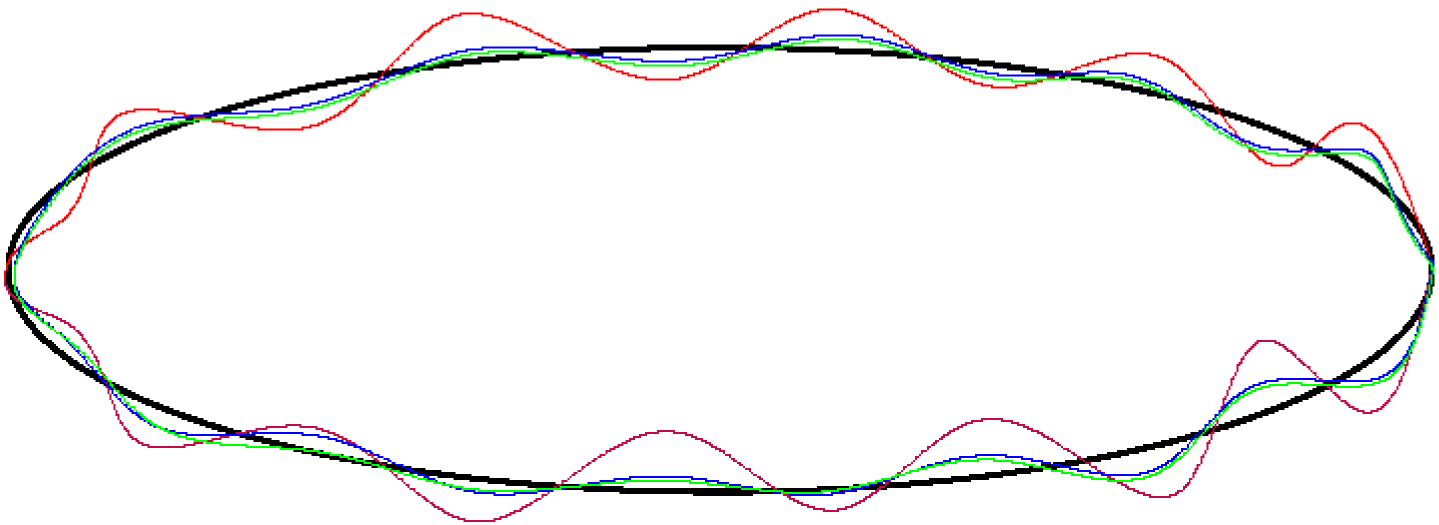
Keplerian ellipse: basic approximation



First approximation (red curve)



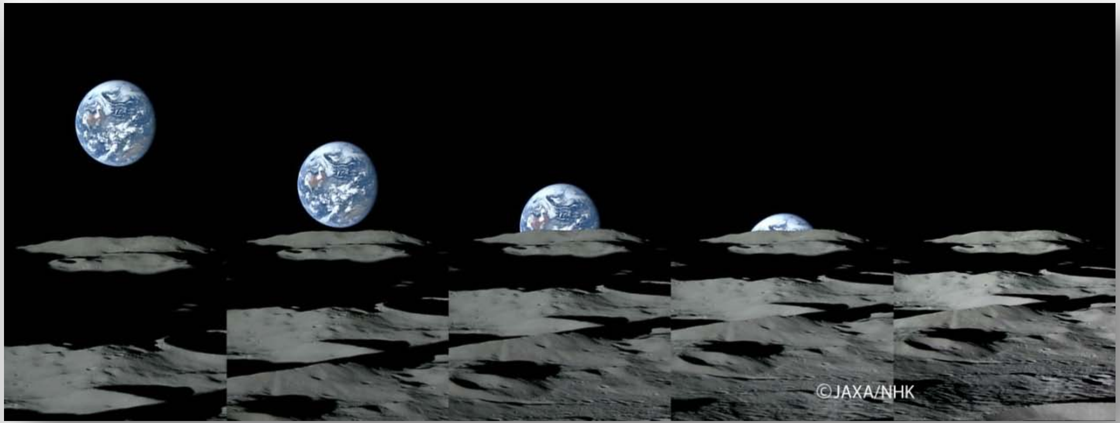
Second approximation (green curve)

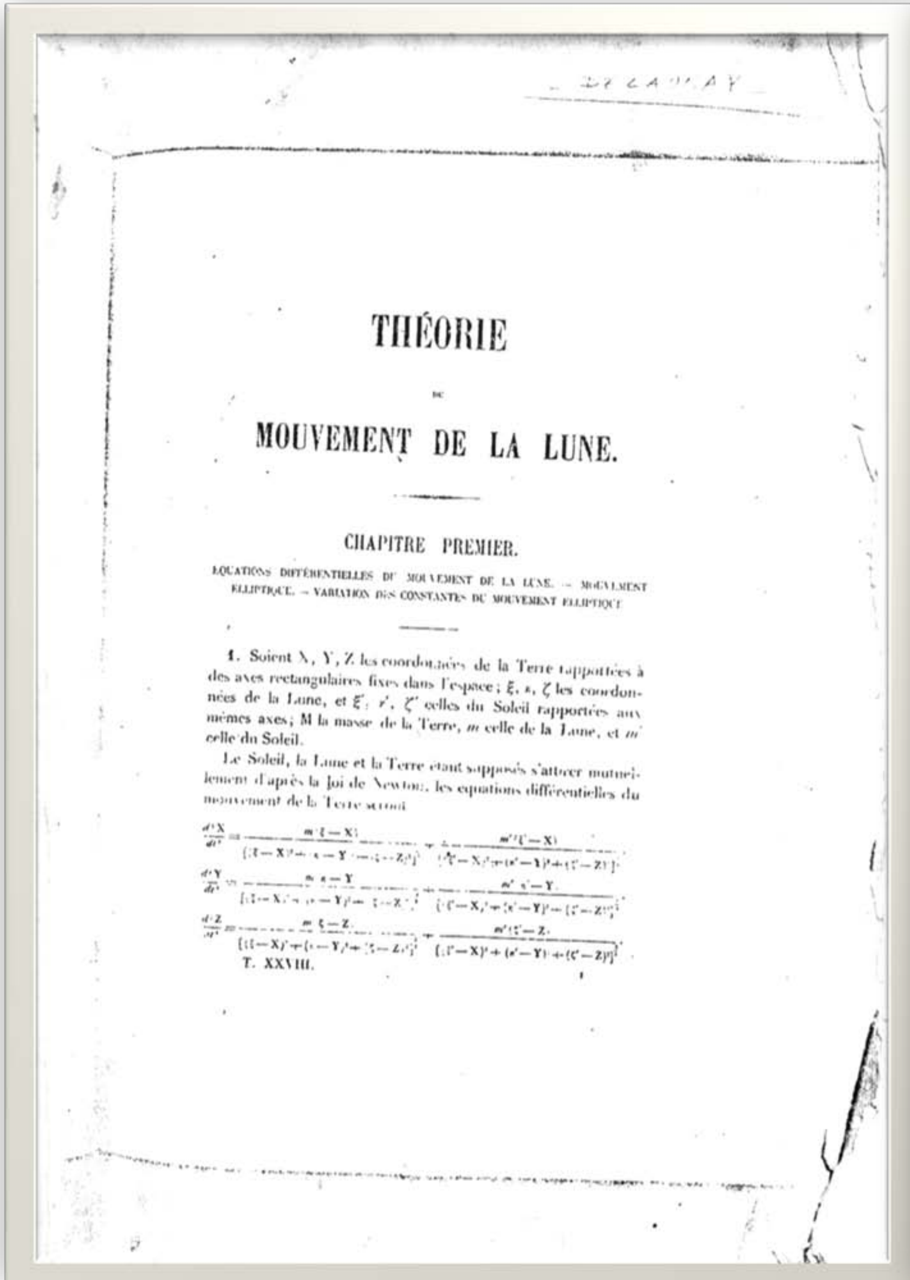


Third approximation (blue curve)

- Perturbation theory allows to determined an approximate solution of the equations of motion (Laplace, Lagrange, Delaunay, Leverrier, etc., XVIII-XIX century).

- Charles Delaunay (1816-1872) developed a very precise lunar motion based on perturbation theory.





DELAUNAY

THÉORIE DU MOUVEMENT DE LA LUNE.

CHAPITRE PREMIER.

ÉQUATIONS DIFFÉRENTIELLES DU MOUVEMENT DE LA LUNE. — MOUVEMENT ELLIPTIQUE. — VARIATION DES CONSTANTES DU MOUVEMENT ELLIPTIQUE.

1. Soient X, Y, Z les coordonnées de la Terre rapportées à des axes rectangulaires fixes dans l'espace; ξ, η, ζ les coordonnées de la Lune, et ξ', η', ζ' celles du Soleil rapportées aux mêmes axes; M la masse de la Terre, m celle de la Lune, et m' celle du Soleil.

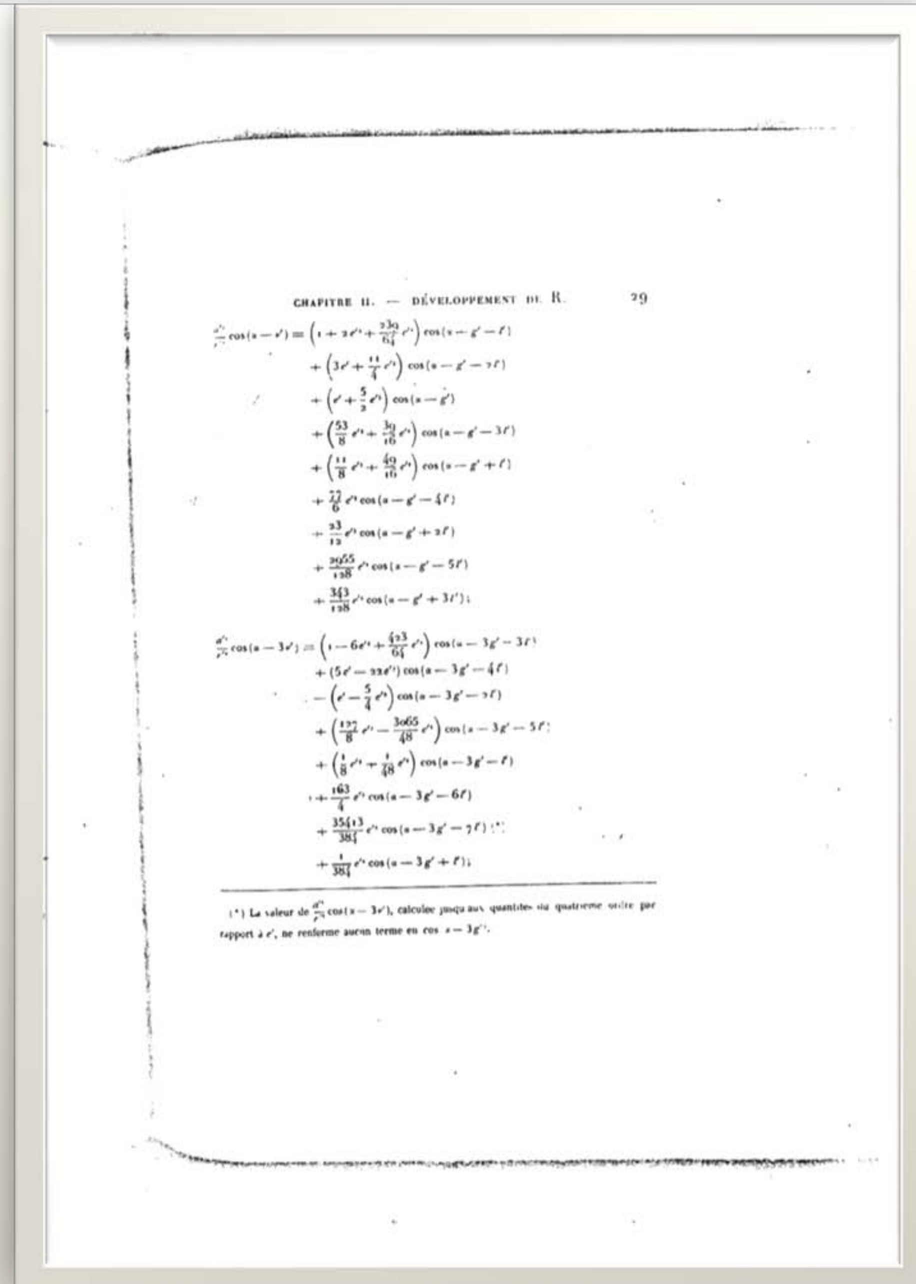
Le Soleil, la Lune et la Terre étant supposés s'attirer mutuellement d'après la loi de Newton, les équations différentielles du mouvement de la Terre seront

$$\frac{d^2X}{dt^2} = \frac{m(\xi - X)}{[(\xi - X)^2 + (\eta - Y)^2 + (\zeta - Z)^2]^{\frac{3}{2}}} + \frac{m'(\xi' - X)}{[(\xi' - X)^2 + (\eta' - Y)^2 + (\zeta' - Z)^2]^{\frac{3}{2}}},$$

$$\frac{d^2Y}{dt^2} = \frac{m(\eta - Y)}{[(\xi - X)^2 + (\eta - Y)^2 + (\zeta - Z)^2]^{\frac{3}{2}}} + \frac{m'(\eta' - Y)}{[(\xi' - X)^2 + (\eta' - Y)^2 + (\zeta' - Z)^2]^{\frac{3}{2}}},$$

$$\frac{d^2Z}{dt^2} = \frac{m(\zeta - Z)}{[(\xi - X)^2 + (\eta - Y)^2 + (\zeta - Z)^2]^{\frac{3}{2}}} + \frac{m'(\zeta' - Z)}{[(\xi' - X)^2 + (\eta' - Y)^2 + (\zeta' - Z)^2]^{\frac{3}{2}}}.$$

T. XXVIII. 1



CHAPITRE II. — DÉVELOPPEMENT DE R. 29

$$\frac{a'^3}{r'^3}\cos(u - v') = \left(1 + 2e'^2 + \frac{239}{64}e'^4\right)\cos(u - g' - l')$$
$$+ \left(3e' + \frac{11}{4}e'^3\right)\cos(u - g' - 2l')$$
$$+ \left(e' + \frac{5}{2}e'^3\right)\cos(u - g')$$
$$+ \left(\frac{53}{8}e'^2 + \frac{39}{16}e'^4\right)\cos(u - g' - 3l')$$
$$+ \left(\frac{11}{8}e'^2 + \frac{49}{16}e'^4\right)\cos(u - g' + l')$$
$$+ \frac{77}{6}e'^3\cos(u - g' - 4l')$$
$$+ \frac{23}{12}e'^3\cos(u - g' + 2l')$$
$$+ \frac{2955}{128}e'^4\cos(u - g' - 5l')$$
$$+ \frac{343}{128}e'^4\cos(u - g' + 3l');$$

$$\frac{a'^3}{r'^3}\cos(u - 3v') = \left(1 - 6e'^2 + \frac{423}{64}e'^4\right)\cos(u - 3g' - 3l')$$
$$+ (5e' - 22e'^3)\cos(u - 3g' - 4l')$$
$$- \left(e' - \frac{5}{4}e'^3\right)\cos(u - 3g' - 2l')$$
$$+ \left(\frac{127}{8}e'^2 - \frac{3065}{48}e'^4\right)\cos(u - 3g' - 5l')$$
$$+ \left(\frac{1}{8}e'^2 + \frac{1}{48}e'^4\right)\cos(u - 3g' - l')$$
$$+ \frac{163}{4}e'^3\cos(u - 3g' - 6l')$$
$$+ \frac{35413}{384}e'^4\cos(u - 3g' - 7l') \text{ (*)}$$
$$+ \frac{1}{384}e'^4\cos(u - 3g' + l');$$

(*) La valeur de $\frac{a'^3}{r'^3}\cos(u - 3v')$, calculée jusqu'aux quantités du quatrième ordre par rapport à e', ne renferme aucun terme en $\cos(u - 3g')$.

"Theorie du Mouvement de la Lune"
C. Delaunay

Preliminary computations

auquel on aurait dû s'arrêter, d'après ce qui vient d'être dit, et cela pour des raisons spéciales qui seront indiquées plus tard (chapitre IV).

Ajoutons encore que, e' étant environ trois fois plus petit que γ et e, dans le rejet des termes d'un ordre supérieur à celui auquel on voulait s'arrêter, on a regardé e'^2, e'^3, e'^4, comme des quantités des quatrième, cinquième, sixième ordres; e'^4 comme une quantité du huitième ordre, etc.

En opérant conformément aux explications qui précèdent, on a trouvé pour R la valeur suivante :

$R = \frac{\mu}{2a}$ [illegible]

[illegible]

[illegible]

The function starts at page 33…

… continues at p. 34, 35, 36, 37 ….

[illegible]

[illegible]

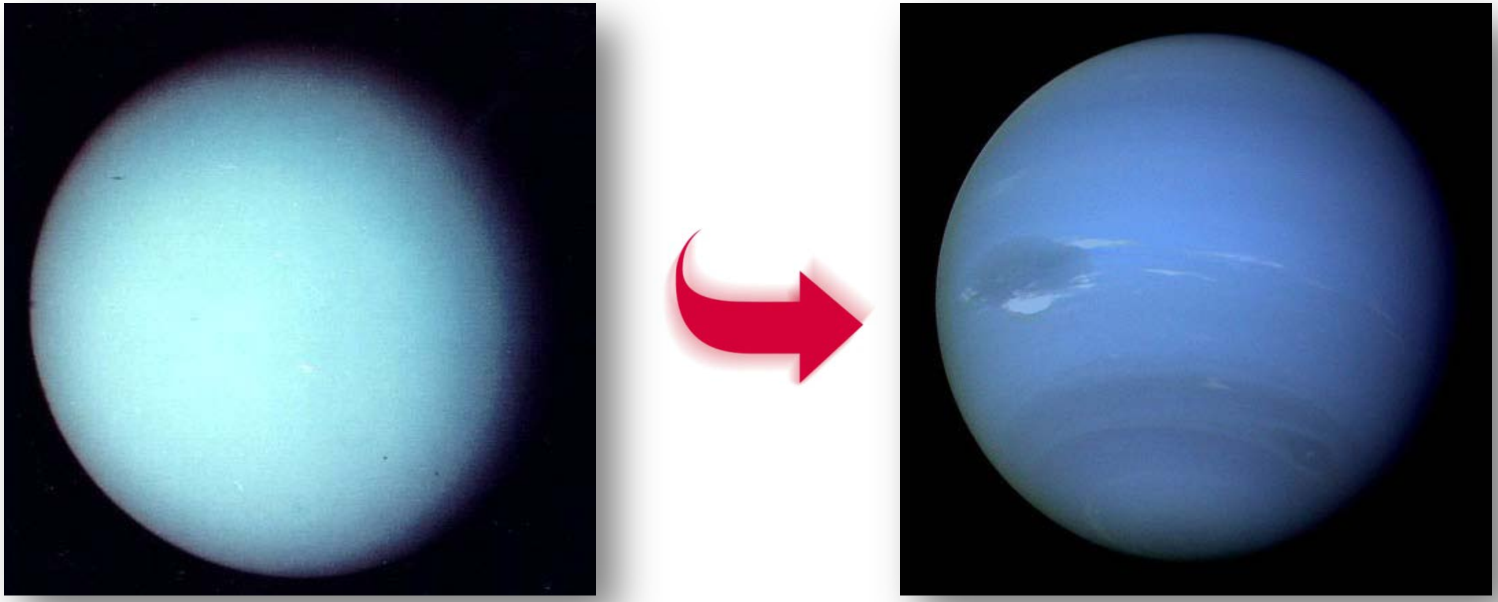
15. Au moyen du développement de R qui vient d'être donné, on pourra déterminer les valeurs de L, G, H, l, g, h, en fonction du temps, en se servant des équations (9). Les valeurs de ces six quantités devront ensuite être substituées dans les expressions des coordonnées de la Lune, ce qui donnera définitivement ces coordonnées en fonction du temps.

… again at page 38, 39, 40, 41 …

… stops at page 54!

- Neptune was discovered by Leverrier (1811-1877) and Adams (1819-1892) using perturbation theory, due to anomalies observed in the motion of Uranus.

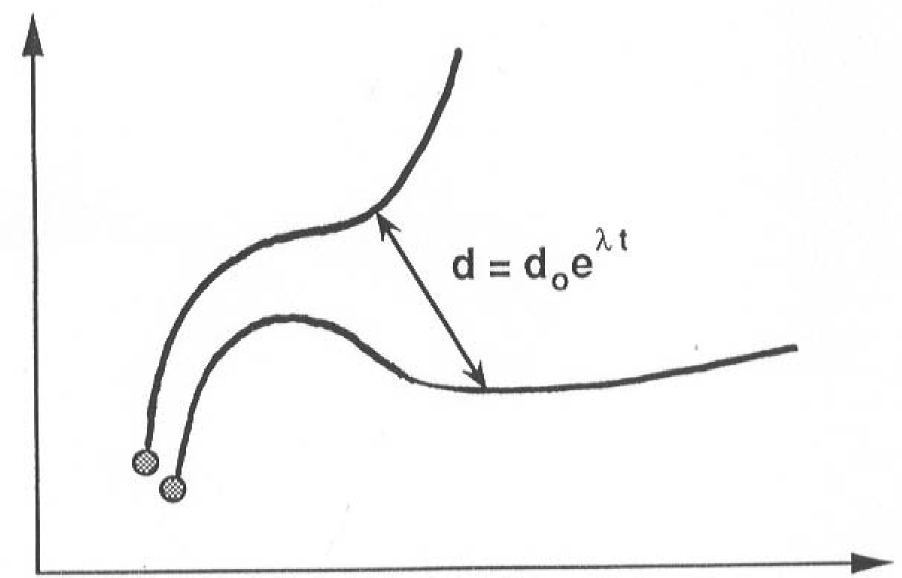
- What happens to the long-term stability of the planets?



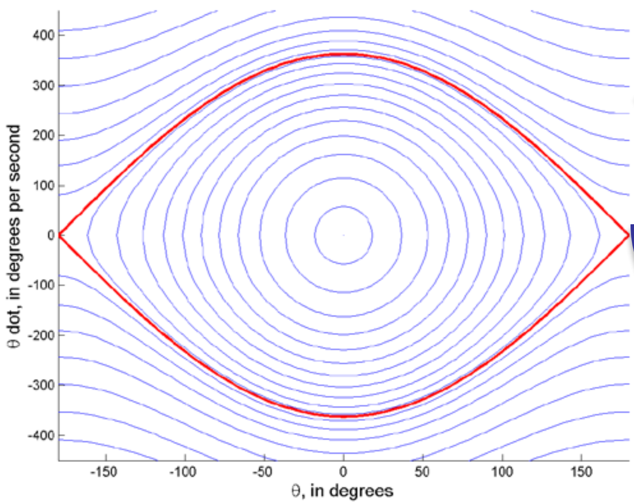
Images: PD-USGOV-NASA, NASA

7. Chaos

- Chaos: irregular motion showing an *extreme sensitivity to the choice of the initial conditions*.
- Poincaré: discovered chaos while studying the 3-body problem (later Lorenz in 1962 the "Butterfly Effect").
- *Chaos does not mean that a system is unstable, but rather unpredictable.*

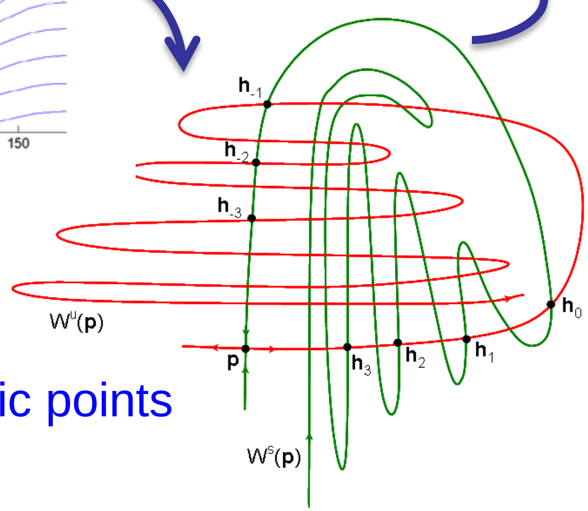


- Earth-Moon-spacecraft= 3-body pb, no Kepler laws
- Poincaré: 3-body problem, homoclinic points, chaos
- Kolmogorov: KAM theory, regular orbits

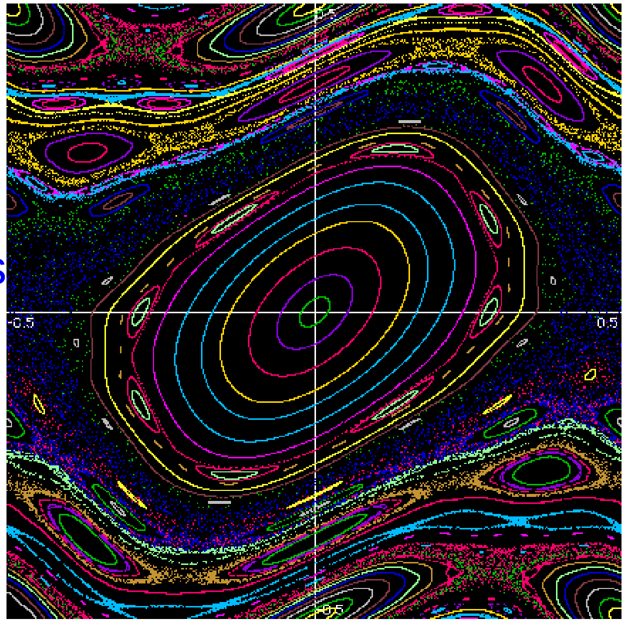


pendulum:
unstable point

chaos



homoclinic points



8. Resonances

RESONANCE

Orbital
(mean motion)

Involving 3 objects
(Sun, Jupiter, Saturn)

Relation between
orbital periods

Spin-orbit

Involving 2 objects
(Earth. Moon)

Between orbital and
Rotational periods

9. Orbital resonances

- 3 bodies: S (Sun), A (asteroid), J (Jupiter)
- Let T_A e T_J be the periods of revolution around S.

- **Definition**: An *orbital resonance* between A e J occurs when:

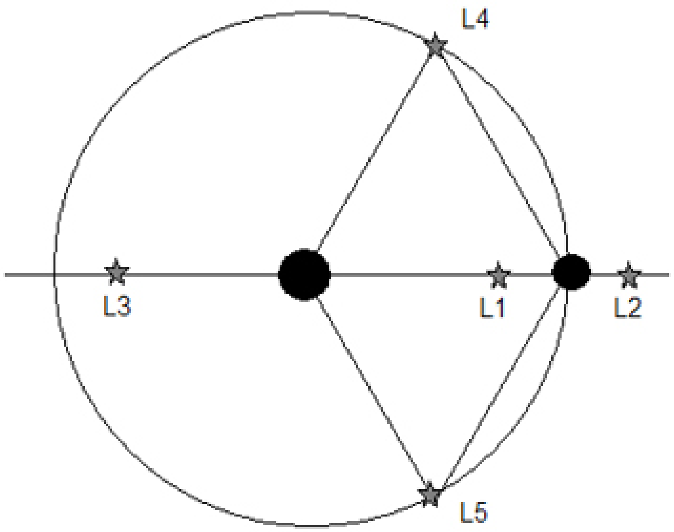
$$T_A / T_J = p/q \quad \text{with p,q non-zero intergers.}$$

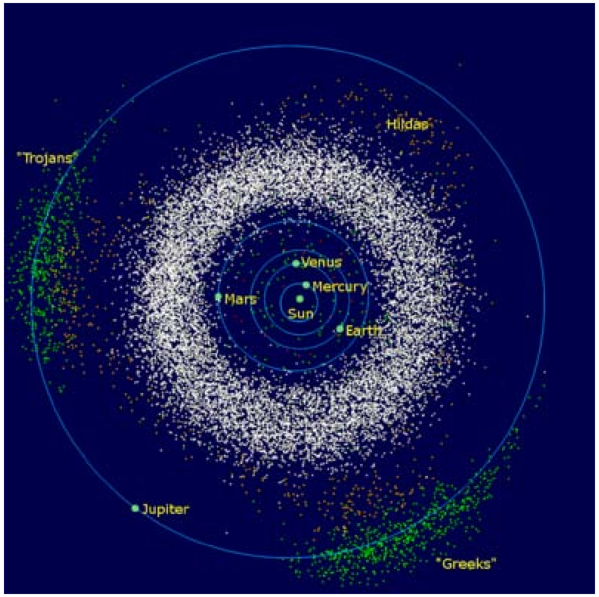
- Examples:
- Jupiter and Saturn: $T_J / T_S = 2/5$ or 2 Saturn's orbits correspond to 5 Jupiter's orbits;
- Io, Europa, Ganimede, Callisto: $T_{IO}/T_{EUR}= 1/2$, $T_{IO}/T_{GAN}=1/4$, $T_{EUR}/T_{GAN}=1/2$;
- Satellites of Saturn: $T_{Titan}/T_{Hyperion}=3/4$, $T_{Titan}/T_{Japetus}=1/5$;
- Greek and Trojan asteroids 1/1.

10. Greek and Trojan asteroids

- Two groups of asteroids in 1:1 resonance with Jupiter (same orbital period, same distance from the Sun).

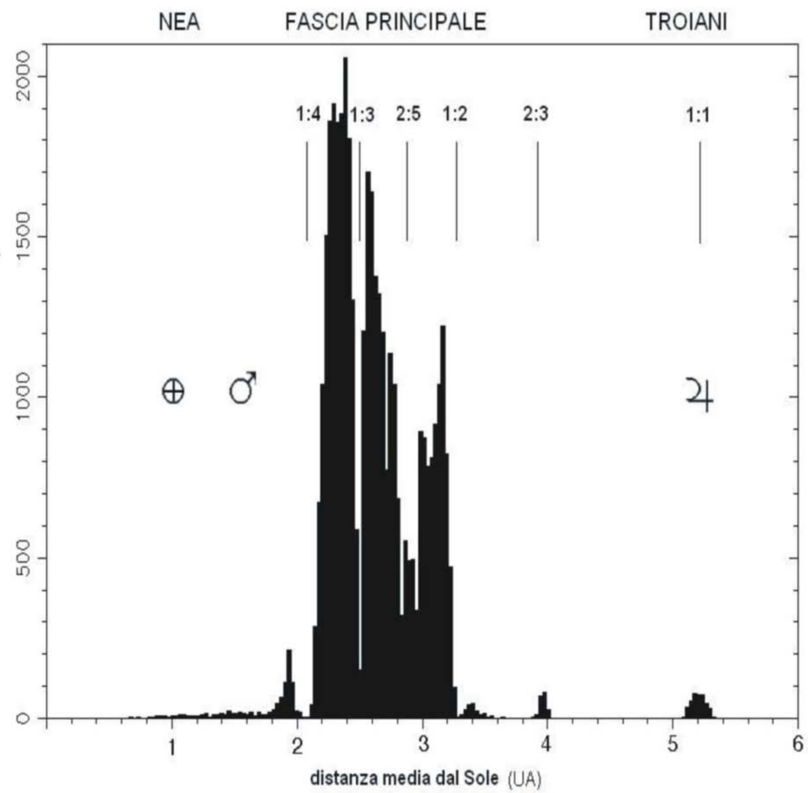
- Euler collinear points L1, L2, L3; Lagrange triangular points L4, L5 (Greek and Trojans).





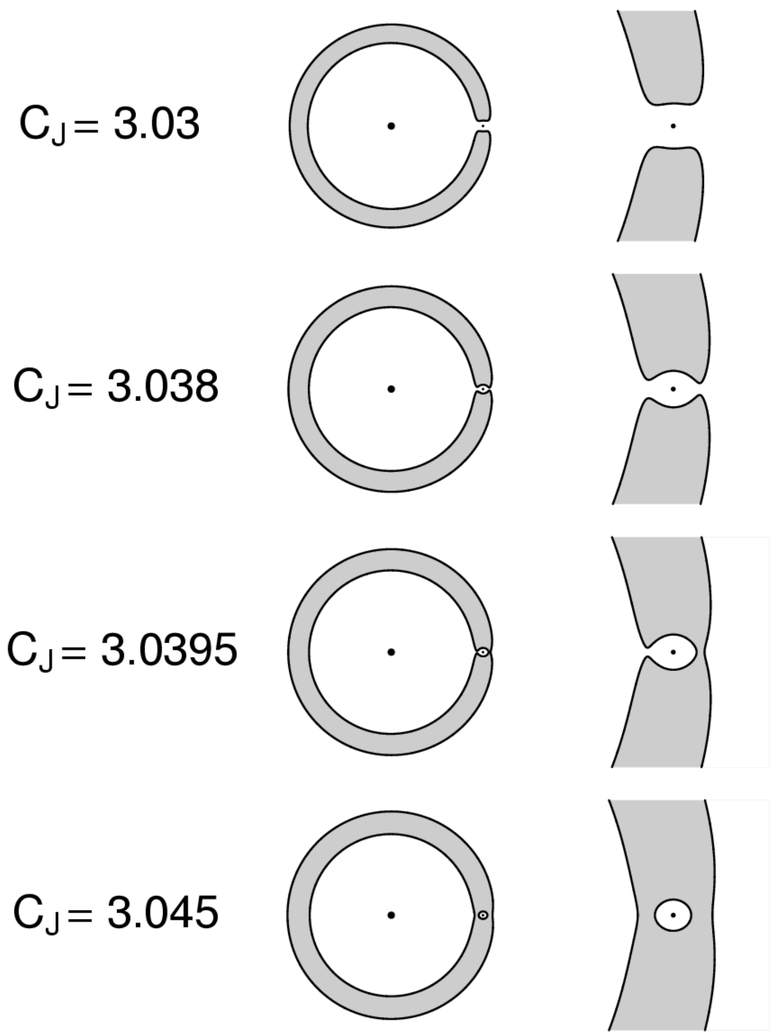
11. Full and empty resonances

- Main belt asteroids between Mars and Jupiter: some resonances are full (1:1, 2:3), other regions called *Kirkwood gaps are empty* (1:2, 1:3, 1:4).



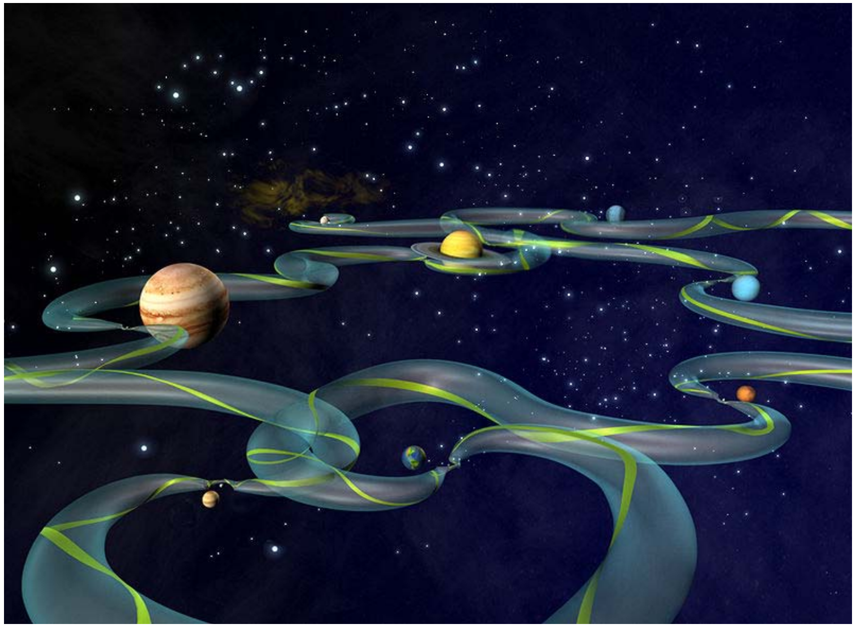
12. Interplanetary highways

- J.-L. Lagrange: Cette recherche n'est à la vérité que de pure curiosité
- C. Conley (1968): use the bottleneck between the primaries and chaos around the collinear points to travel at low cost (use Moser's version of Lyapunov theorem).



Conley: "*Unfortunately, orbits such as these require a long time to complete a cycle (e.g., 6 months, though a modification of the notion might improve that). On the other hand, one cannot predict how knowledge will be applied – only that it often is*".

- International Sun/Earth Explorer 3 (ISEE-3) 1978
- SOHO (1995)
- MAP (2001)
- GENESIS (2001)
- HERSCHEL-PLANCK (2009)



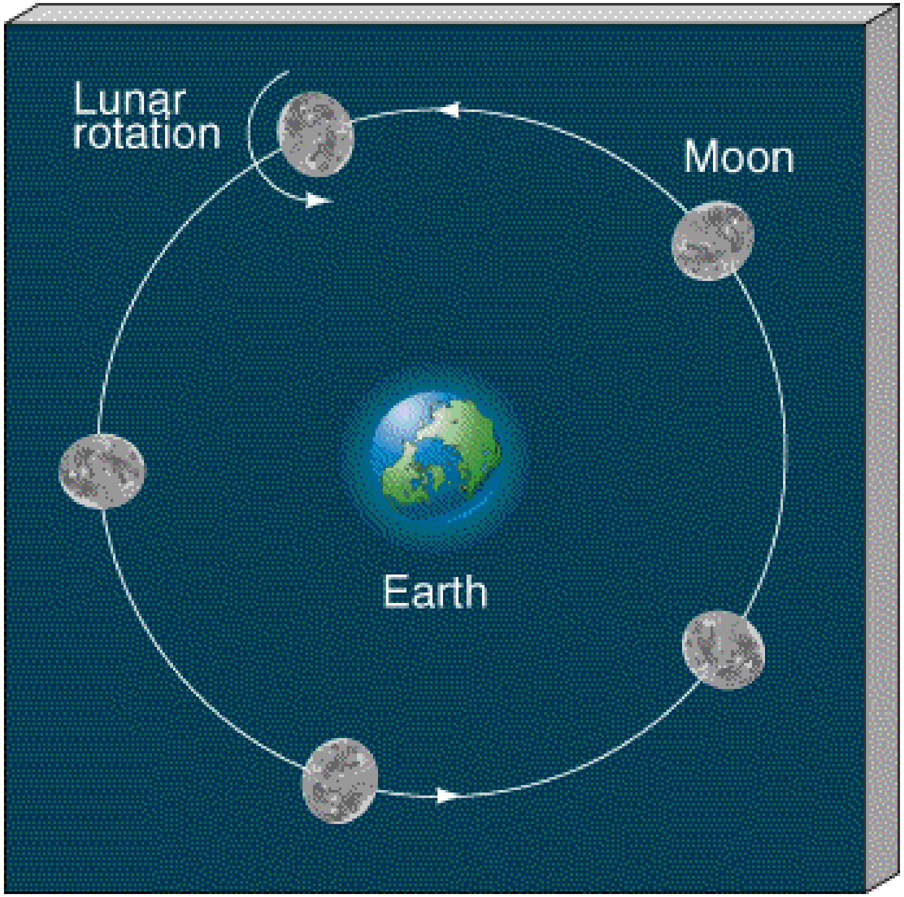
13. Spin-orbit resonances

- Earth and Moon with masses m_E e m_M; T_{rev} orbital period of M around E and T_{rot} rotational period of M (rigid body) around an internal spin axis.

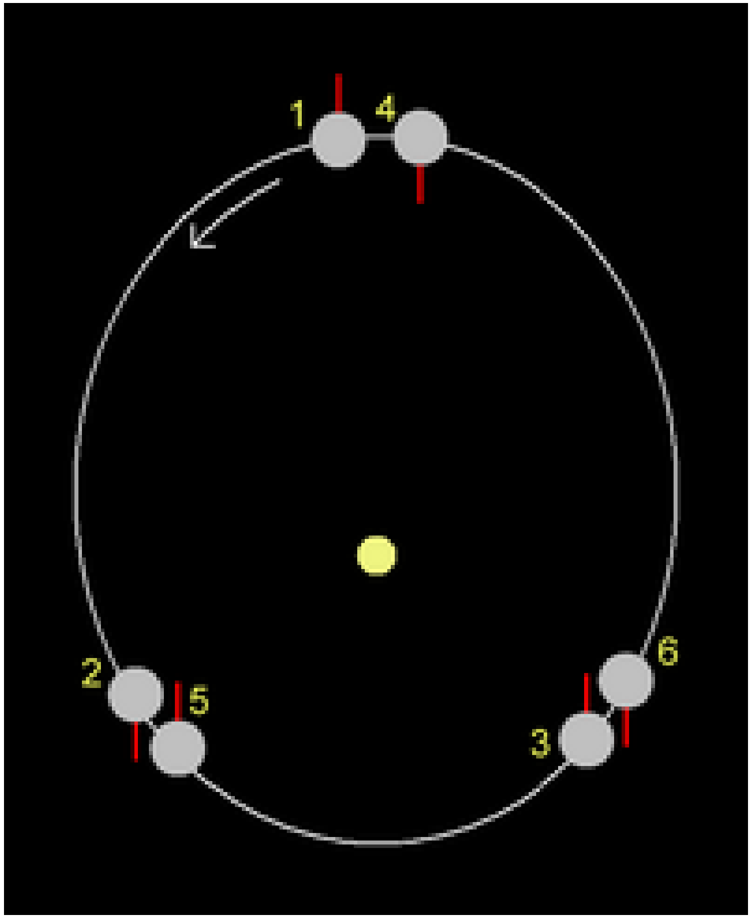
- **Definition**: A *spin-orbit resonance* of order p/q, occurs if

$$T_{rev} / T_{rot} = p/q \text{ with } p,q \text{ non-zero integers.}$$

- Most famous example: 1/1 Earth-Moon spin-orbit resonance, where the Moon always points the same face to the Earth.
- Mercury-Sun: 3/2 spin-orbit resonance, 2 revolutions of Mercury around the Sun correspond to 3 rotations about its spin-axis.
- Hyperion is in chaotic spin-orbit dynamics.

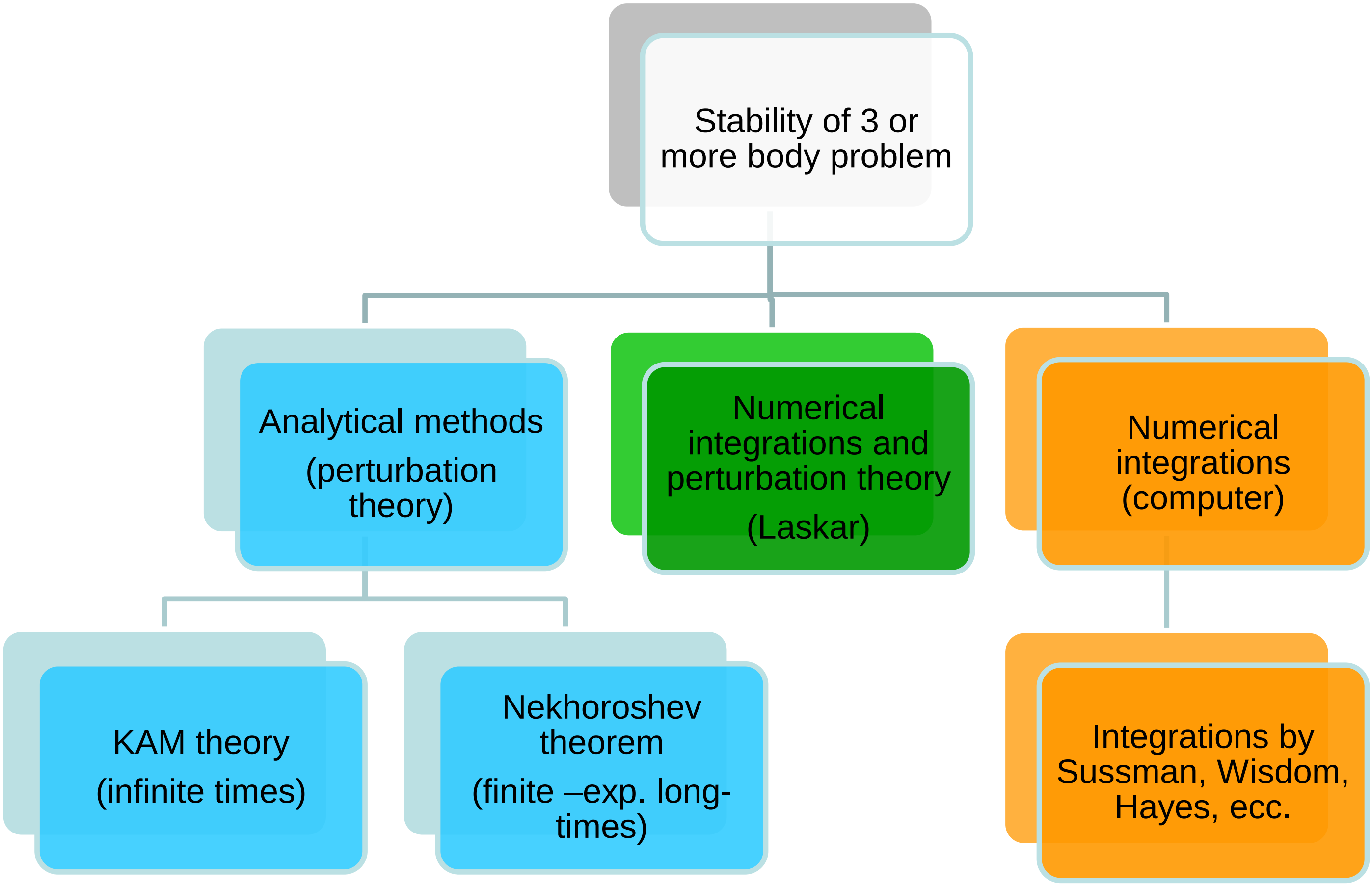


Resonance 1/1



Resonance 3/2

14. Is the Solar system stable?



- Laskar: the internal Solar system is CHAOTIC.
- From an error of 15 mt on the initial position of the Earth: error 150 mt after 10 million years
- error 150 million km after 100 million years, no further predictions!

❑ ***RESULTS:***

- *Mercury and Mars very chaotic*
- *Venus and Earth moderately chaotic*
- *External planets are regular*
- *Pluto very chaotic.*

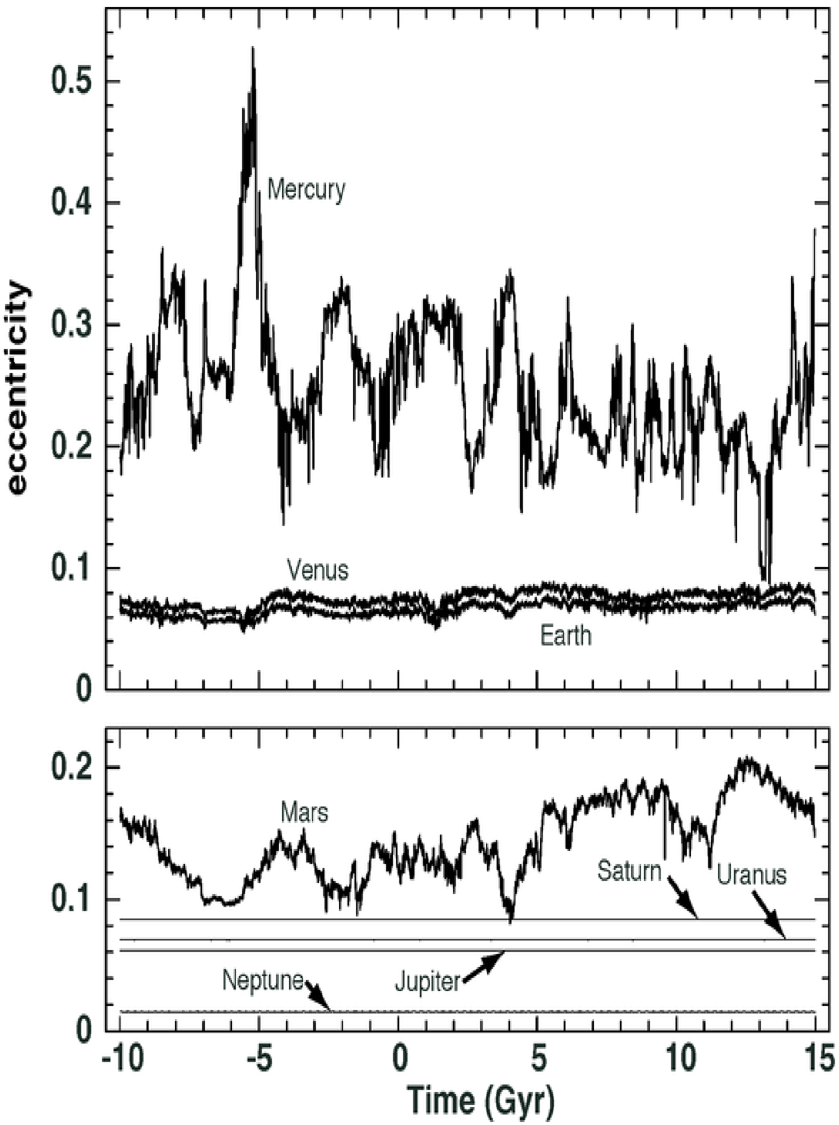


Immagine: J. Laskar, 1994