Abstract of posters

From non-smooth to smooth friction models, using regularisation and slow-fast theory

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Many different phenomena in nature are modelled using non-smooth vector fields, i.e. differential equations with a discontinuous right hand side.

In mechanics, rigid body dynamics with impacts, switching or friction are all examples of non- smooth systems. Other examples may be found in seismology or control theory.

The poster will focus on dry friction and in particular on stick-slip phenomena. The understanding of the behaviour of dry friction is of key importance in several industrial sectors, for instance in brake manufacturing.

Several different friction models are present in literature and all of them are non-smooth. We are interested to see whether regularisation together with the use of slow-fast theory may be of use towards the unification of all the models. We also believe that we may get more insight on the brake squealing phenomenon by studying the regularised friction model.

When considering the friction interaction at a micro-scale level, we obtain in general smooth models. Thus we are interested to understand whether the non-smoothness of the models is eventually an idealisation of a smooth phenomenon.

We show that the widely used single degree of freedom oscillator describing the brake-pad interaction is not robust respect to regularisation. Indeed a canard solution may appear under certain conditions. The canard is a solution of singularly perturbed systems with a rather surprising behaviour. Indeed, it follows an attracting slow manifold, passes close to a bifurcation point of the critical manifold and then follows a repelling slow manifold for a considerable amount of time.

We also consider rate and state models, mainly used to describe fault dynamics in earthquakes. In these models, the state variable describes the aging process of the surface interaction due to friction. An analysis of the model is performed, when rewritten in a slow-fast formulation.

The reversibility problem for dynamical systems

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We will give necessary conditions for the orbital-reversibility for a class of planar dynamical systems. Based in these conditions, we formulate a suitable algorithm to detect orbital-reversibility which is applied to a family of nilpotent systems and to a family of degenerate systems. More concretely, we consider a planar autonomous system of differential equations having an equilibrium point at the origin given by

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x, y)^T \in \mathbb{R}^2$. We study if it admits some reversibility modulo $\mathcal{C}^\infty$-equivalence.

The problem of determining if system (1) has some reversibility is consider in [1] and [3]. In [2], we are concerned with the *orbital-reversibility* problem: a system is called *orbital-reversible* if there exists some time-reparametrization such that the resulting system admits some reversibility; and our goal is to determine, in the planar case, conditions on the system to be orbital?reversible. As with the reversibility, the presence of some orbital?reversibility is useful in the understanding of the dynamical behavior of the system, because the time-reparametrizations do not change the orbits but only the speed in which they are traversed in time. For planar systems, there is a strong connection between the center problem and the reversibility property of a planar system: if the system has a non-degenerate center at the origin, then it is reversible, see [5]. The *orbital-reversibility* property is also closely related to the center problem. For instance, the existence of an orbital reversibility in a monodromic vector field ensures the presence of a center. Also, if a planar system has a nilpotent center at the origin, then it is orbital-reversible, see [4].

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Dynamics and bifurcations in 3D Filippov Systems at a Degenerate T-singularity

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We consider the Filippov systems with two zones in $\mathbb{R}^3$ separated by a switching manifold S, where the dynamics in each zone is governed by a different smooth vector field. We define such vector fields by $\mathbf{F}^-$ and $\mathbf{F}^+$, and by S^- and S^+ their respective sets of tangency points with S. When the sets S^- and S^+ transversely intersect at a point where both vector fields $\mathbf{F}^-$ and $\mathbf{F}^+$ has invisible quadratic tangency with S, we call this point of T-singularity (Teixeira singularity). The degenerate T-singularity occurs when the sets S^- and S^+ intersect non-transversely.

In this work we analyze the local dynamics at the T-singularity points in the cases where the contact between S^- and S^+ is quadratic and when is transversal. Furthermore, we analyze the bifurcations involving the annihilation (or generation) of one of the regions of different dynamic behaviors on S (crossing, sliding or escaping) along with the disappearance (or birth) of T-singularity points.

Masses and Vortices Dynamics on a Cylinder

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The goal of this study is to formulate and to understand some of the features of the dynamics of masses and vortices on a cylinder. Considering the intrinsic geometry of the cylinder, the masses potential and the vortices hamiltonian are derived, which gives us the equations of the dynamics. We consider, separately, the masses and the vortices as points on the cylinder. Some reductions can be done, in order to better understand this dynamics. For the mass dynamics we focused on the case of two masses. After that, some numerical experiments were done. In particular, some Poincaré sections were plotted to gather some information about this dynamics.

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Towards the analysis of a canonical form for PWL systems with hysteresis

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We consider piecewise linear systems in $\mathbb{R}^3$, with symmetry with respect to the origin and with slow-fast dynamics. Assuming that the transition times of the fast dynamics are negligible, we get a planar system where the only non-linearity is represented by the hysteresis function.

Starting from the simplest situations, work is in progress to analyse the existence of possible periodic orbits and their bifurcations. In particular, we consider the focus-focus configuration, which is known to be capable of exhibiting chaotic behaviour. To get a gradual understanding of its dynamical richness, we start by focussing on a critical choice of parameters characterized by the existence of orbits that are simultaneously tangent to the two jump straight lines, showing the coexistence of 4 different periodic orbits.

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Inverse integrating factor for degenerated vector fields

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In this work, we study the existence of an inverse integrating factor for a class of systems, in general non-analytically integrable, whose lowest-degree quasi-homogeneous term is a Hamiltonian system and its Hamiltonian function only has simple factors over $\mathrm{C}[x, y]$. That is, we deal with systems of the form

$$\dot{\mathbf{x}} = \mathbf{X}_h + \text{q-h.h.o.t.} \tag{2}$$

For this task, firstly, we calculate a formal orbital equivalent normal form of system (2), i.e. an expression of this system after a change of state variables and a reparameterization of the time, and we focus our study in systems (2) which are formally orbital equivalent systems to

$$\dot{\mathbf{x}} = \mathbf{X}_h + \mu \mathbf{D}_0, \text{ with } \mu = \sum_{j>r} \mu_j, \ \mu_j \in \mathrm{Cor}(\ell_j) \text{ and } h \in \mathcal{P}^{\mathbf{t}}_{r+|\mathbf{t}|} \tag{3}$$

where ℓ_j is the Lie derivative of the lowest degree quasi-homogeneous term of (2).

They are a wide class of these systems, for example, systems with linear part non-null (such as nilpotent systems) and some generalized nilpotent, among others. From Algaba et al. [1], systems formally orbital equivalent to systems (3) are analytically integrable if and only if $\mu \equiv 0$ and, in such a case, they have a first integral of the form $h +$ q-h.h.o.t., and consequently, they have an inverse integrating factor. Therefore, for $\mu \not\equiv 0$, systems orbitally equivalent to systems (3) do not have any analytic first integral (non-integrable systems). We study this systems, which are non-integrable, and we show our principal result. In it we give necessary and sufficients conditions for the existence of an inverse integrating factor for systems (3) and we apply this result for the characterization the existence of an inverse integrating factor for some families of generalized nilpotent systems (see [2]). Other works in this way can be seen in Algaba et al. [3, 4].

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Arnold diffusion using several combinations of Scattering maps

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In this work we illustrate the Arnold diffusion in a concrete example - a priori unstable Hamiltonian system of 2+1/2 degrees of freedom $H(p,q,I,\varphi,s) = \frac{p^2}{2} + \cos q - 1 + \frac{I^2}{2} + h(q,\varphi,s;\varepsilon)$ - proving that for any small periodic perturbation of the form $h(q,\varphi,s;\varepsilon) = \varepsilon \cos q(a_{00} + a_{10}\cos\varphi + a_{01}\cos s)$ ($a_{10}a_{01} \neq 0$ and $\varepsilon \neq 0$ small enough) there is the global instability for the action, i.e., $I(0) \leq -I(\varepsilon) < I(\varepsilon) \leq I(T)$ for some T and $0 < I(\varepsilon) \leq C \log\frac{1}{\varepsilon}$ for some constant C. For this, we apply a geometrical mechanism: based in the so-called the Scattering map.

This work has the following structure: In a first stage, for the more restricted case ($I(\varepsilon) \sim \pi/2\mu$, $\mu = \frac{a_{10}}{a_{01}}$), we use only one Scattering map. Later, for general case we combine a Scattering map and the inner map (inner dynamics) to prove the main result (the existence of the instability for any μ). Finally, we consider several scattering maps and we show several "ways of diffusion" using multiple combinations of scattering maps.

A fractalization route for affine skew-products on the complex plane

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Recall that Floquet Theory states that a linear ordinary differential equation with periodic coefficients can be reduced, by means of a periodic change of variables, to constant coefficients. The question of when a linear system, induced by a map or by an ODE, is reducible or not becomes much more interesting in the quasi-periodic setting, where there are non-reducible systems. Besides providing criteria to contradistinguish reducible and non-reducible systems, it is also of interest to see how non-reducibility manifests in dynamics.

In this work we study a special kind of maps, the so-called quasi-periodic skew-products on the complex plane. We show that, in this situation, the only source of non-reducibility comes from a topological obstruction. We also study affine systems, the simplest systems in which one observes invariant curves. We show that non-reducibility has a visible impact in the bifurcation at zero Lyapunov exponent, that is, when the curves passes from attracting to repelling. We prove for some cases, that, when its linear behaviour is not reducible, and as the Lyapunov exponent goes to zero, the invariant curve gets destroyed by a mechanism of fractalization.

Dynamical compactness and sensitivity

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We will introduce dynamically compact systems as a new concept of chaotic dynamical systems, given by a compact metric space and a continuous surjective self-map. Observe that any weakly mixing system is transitive compact, and we show that any transitive compact M-system is weakly mixing. Then we will discuss the relationships among it and other several stronger forms of sensitivity. In particular, we will prove that any transitive compact system is Li-Yorke sensitive and furthermore multi-sensitive if it is not proximal.

Quasideterminant solutions of NC Painlevé II equation with the Toda solution at $n = 1$ as a seed solution in its Darboux transformation

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Here I have presented the derivation of Darboux transformations for the noncommuting elements φ and ψ of noncommutative Toda system at $n = 1$ with the help of zero curvature representation to the associated systems of non-linear differential equations. I also derive the quasideterminant solutions to the noncommutative Painlevé II equation by taking the Toda solutions at $n = 1$ as a seed solution in its Darboux transformations. Further by iteration, I generalize the Darboux transformations of these solutions to the Nth form.

A Study on Temperature Distribution, Efficiency and Effectiveness of longitudinal porous fins by using Adomian Decomposition Sumudu Transform Method

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Here, we studied the variation of temperature distribution, efficiency and effectiveness of porous fin for different fractional order α, porous parameter ξ and convection parameter δ by using Adomian Decomposition Sumudu Transform Method (ADSTM). Here the geometry considered is that rectangular porous fin and the passage velocity in heat transfer through porous media is simulated by using Darcy's model.

The role of isochrons in neural communication

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Neural activity in the brain is able to generate rhythmic patterns. In our poster, we will study how frequency or amplitude of a population whose activity is oscillating can be affected when an external perturbation (i.e the activity of an external neural population) is applied. For this aim we will use a Wilson-Cowan rate model. Our results show how different resonances between perturbation and network frequencies appear. We also found the presence of bistable solutions.

Odd Global Continuation and Linear Stability of the equilibrium solution for Nonlinear Oscillators of Pendulum type

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In the sixties, W. Loud obtained interesting results of continuation on periodic solutions in driven nonlinear oscillators with small parameter [3]. In this work Loud's results are extended out for periodically driven Duffing equations with odd symmetry quantifying the continuation parameter for a periodic odd solution which is elliptic and emanates from the equilibrium of the non perturbed problem.

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