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Inivited Article

# Orbital mechanics about small bodies ${ }^{2}$ 

D.J. Scheeres<br>A. Richard Seebass Endowed Chair Professor, Department of Aerospace Engineering Sciences, University Colorado at Boulder, 429 UCB, Boulder, CO 80309, United States

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#### Abstract

Small solar system bodies such as asteroids and comets are of significant interest for both scientific and human exploration missions. However, their orbital environments are among the most highly perturbed and extreme environments found in the solar system. Uncontrolled trajectories are highly unstable in general and may either impact or escape in timespans of hours to days. Even with active control, the chaotic nature of motion about these bodies can effectively randomize a trajectory within a few orbits, creating fundamental difficulties for the navigation of spacecraft in these environments. In response to these challenges our research has identified robust and stable orbit solutions and mission designs across the whole range of small body sizes and spin states that are of interest for scientific and human exploration. This talk will describe the challenges of exploring small bodies and present the practical solutions that have been discovered which enable their exploration across the range of small body types and sizes.


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## 1. Introduction

The scientific exploration of small bodies such as asteroids and comets has become a major component of the world's space robotic fleets, and are poised to become of sincere interest to human exploration as well. The motivation for the study of these bodies are several, and mainly include the fact that they preserve the conditions that were present at the formation of the solar system and can also serve as a detailed record of the evolution of the solar system up to the present time. By studying these pristine relics from earlier epochs of the solar system we understand our own existence. Development of an improved understanding of small bodies also impacts the future of humanity, as asteroids and comets also prove the greatest threat to future civilizations through the impact hazard. Thus, motivations for the study of these bodies abound.

[^0]While there has been much study and analysis of interplanetary trajectories to these small bodies, the real technical challenges emerge once close proximity operations are considered. The small body environment is perhaps the most strongly perturbed astrodynamic environment found in the solar system. A combination of factors create these challenges, including strongly distended body shapes, a range of spin states and rates, the strength of solar perturbations from gravity and radiation pressure, and non-gravitational forces from the bodies themselves in the case of comets. Due to these effects in isolation or combination it is possible for seemingly stable orbits to impact or escape from these bodies in a matter of a few orbits, causing severe constraints on the remote operation of vehicles in these environments. Despite this complexity, the application of methods from astrodynamics, celestial mechanics and dynamical astronomy can be used to find practical mission operations plans and designs across the spectrum of body sizes, types and locations.

There has been significant research on this challenging problem over the last 20 years, resulting in a rich set of papers published on many aspects of these problems, and
culminating in a soon to be released book on the topic [21]. In this paper some of the main issues related to satellite motion in the vicinity of small bodies are reviewed, focusing on the major perturbations arising from the body shape, spin state and solar influence. This paper only focuses on orbital approaches to these issues, although it must be acknowledged that in many circumstances hovering [14,2] or flyby [30] approaches may be more relevant to the mission design goals. This paper is also restricted to "solitary" bodies and does not discuss the many interesting aspects of mission design about binary asteroids [23,1].

Given the orbital mechanics focus, this paper reviews the issues at play and the practical mission design


Fig. 1. Stable, escape and impacting orbits at Eros. The plots are in an inertial frame while the attitude of Eros is shown at the initial conditions.
"solutions" for some of these extreme environments. To provide an example for how these environments interact a case study for orbital motion about a pre-rendezvous model of the target of the Rosetta spacecraft, Comet 67P/ Churyumov-Gerasimenko (comet " $67 \mathrm{P} / \mathrm{CG}^{\prime}$ ) is provided. The effect of non-gravitational accelerations due to outgassing are not considered in detail, although these are a significant and difficult topic of relevance for the Rosetta mission $[4,12]$.

## 2. Motivation

To motivate the specific discussion and review in this paper three examples of the extreme results that can be found for small body orbiters are presented. Fig. 1 shows three orbits about the asteroid 433 Eros, accounting only for the gravitational attraction of that strongly distended body. One of these orbits is started at local circular conditions and remains stable and bounded for arbitrary periods of time. Another is shifted by $45^{\circ}$ in phase angle from the initial stable orbit and escapes within two orbits. The third is started at local circular conditions a few kilometers closer to the body and impacts within two orbits-each orbit lasting on the order of 16 h . This range of effects in close proximity to one geometric location is due entirely to interactions with the body's rotating gravity field, and can be completely understood and accounted for using astrodynamics theory [28].

Figs. 2 and 3 show two spacecraft orbits about a small, spherical asteroid with the only perturbation arising from the solar radiation pressure from the sun. Both of these plots are shown in frames rotating with the asteroid's orbit about the sun, on the order of $1^{\circ}$ per day, and thus keeping the solar location fixed. Fig. 2 presents an example of an initially stable orbit which loses its stability and escapes from the asteroid when the asteroid's distance from the sun becomes closer than a limit which can be explicitly predicted. Fig. 3 shows two orbits, started at locally circular conditions a distance of 100 m apart from each other. It can be seen that even across this small


Fig. 2. Escaping orbit due to SRP.


Fig. 3. Escaping/impacting orbit due to SRP.
range of initial conditions, one orbit impacts while the other one escapes. This shows the sensitivity of orbits to modest changes in initial conditions and explicitly shows the challenges that small body orbiters may face. Despite the complexity of these motions, again it is possible to understand them at a deep level and, using astrodynamics theory, develop mission design solutions [22].

## 3. Orbital mechanics

This section briefly reviews the fundamental equations of motion in the small body environment with specific specializations to gravity dominated and solar dominated regimes of motion. The general problem is first stated and then some specific cases are discussed in more detail in following sections.

As in most orbit mechanics problems, the fundamental equations of motion can be most simply stated in an inertial frame. Take a small body-centered frame with an inertially fixed orientation. Assume an attitude matrix $\mathbf{C}$ that takes a vector expressed in the small body-fixed frame and rotates it into the inertial frame. Thus, $\boldsymbol{C}$ is a function of the small body's rotational dynamics and defines its attitude. Then the equations of motion for a satellite in its vicinity can be generally described as
$\ddot{\boldsymbol{r}}=\frac{\partial \mathcal{U}\left(\boldsymbol{C}^{T} \cdot \boldsymbol{r}\right)}{\partial \boldsymbol{r}}+\frac{\partial \mathcal{R}_{S}(\boldsymbol{r}, \boldsymbol{d})}{\partial \boldsymbol{r}}+\frac{\partial \mathcal{R}_{S R P}(\boldsymbol{r}, \boldsymbol{d})}{\partial \boldsymbol{r}}$
where $\boldsymbol{r}$ is the spacecraft position vector relative to the small body center of mass, $\mathcal{U}$ is the gravitational force potential of the body, $\mathcal{R}_{S}$ represents the gravitational perturbation from the sun, and $\mathcal{R}_{S R P}$ represents the solar radiation pressure perturbation. The vector $\boldsymbol{d}$ represents the position of the small body relative to the sun (assumed to follow 2-body motion). The gravitational force potential is most generally defined as
$\mathcal{U}=\mathcal{G} \int_{\mathcal{B}} \frac{d m(\boldsymbol{\rho})}{|\boldsymbol{r}+\boldsymbol{\rho}|}$
where $\mathcal{G}$ is the gravitational constant, $\mathcal{B}$ represents the small body mass distribution, and $\rho$ is the position of a differential mass element $d m$ in the small body fixed frame. The representation of this gravity field is usually performed using spherical harmonic expansions or, when in close proximity to the body, with specialized closedform solutions for a polyhedral-shaped body [31].

The solar gravitational attraction and the solar radiation pressure are represented using simplified models that capture the main aspect of these forces. Higheraccuracy models can be developed, but the essence of the problem arises from these first-order perturbations
$\mathcal{R}_{S}=\frac{\mu_{S}}{2 d^{3}}\left[3(\hat{\boldsymbol{d}} \cdot \boldsymbol{r})^{2}-r^{2}\right]$
$\mathcal{R}_{S R P}=\frac{\beta}{d^{3}} \boldsymbol{d} \cdot \boldsymbol{r}$
where $\mu_{S}$ is the sun's gravitational parameter and $\beta=P_{\Phi}(1+\rho)(A / m)$ is a combination of the sun's radiation flux $P_{\Phi} \sim 1 \times 10^{8} \mathrm{~kg} \mathrm{~km}{ }^{3} / \mathrm{s}^{2} / \mathrm{m}^{2}$, the satellite's reflectance ( $\rho$ ) and the satellite's area to mass ratio in units of $\mathrm{kg} / \mathrm{m}^{2}$. This solar radiation pressure model is commonly referred to as the "cannonball" model, and it suffices to capture the main perturbations from SRP, although improved models have been developed in the literature [10].

To simulate the general motion of a satellite about a small body then requires the specification of that body's gravitational field, $\mathcal{U}$, its rotational dynamics, $\boldsymbol{C}$, its orbit about the sun, $\boldsymbol{d}$, and the satellite area-to-mass ratio and its optical properties, contained in $\beta$. The analytical study of all of these effects in conjunction is difficult, and only limited results are available [22]. However, if these effects are viewed independently, splitting gravitational and solar effects, then significant progress has been made in understanding and analyzing the resultant behavior. These effects are first discussed in isolation, an investigation of what happens when they are combined is given later in this paper, using the Rosetta spacecraft at comet $67 \mathrm{P} / \mathrm{CG}$ as an explicit example.

## 4. Gravity regime

First consider the motion of a satellite in the "gravity dominated" regime, defined as one where the perturbations from the solar gravity and radiation are small compared to the gravitational attraction of the central body. This is the regime that was experienced by the NEAR spacecraft at asteroid Eros, and in general will occur for bodies several kilometers or larger at 1 AU , for example. Under this assumption the only force acting on the satellite is from the rotating gravity field of the small body.

### 4.1. Body-fixed frame analysis

The dynamical properties of motion in such a system are studied by transforming to a body-fixed frame, which removes the attitude matrix $\boldsymbol{C}$ from the equation but introduces the angular velocity vector of the small body $\omega$. A usual assumption is that the body follows torque-free
rotation, which is generally accurate over time spans of interest to a space mission. The equations of motion are then
$\ddot{\boldsymbol{q}}+\dot{\boldsymbol{\omega}} \times \boldsymbol{q}+2 \boldsymbol{\omega} \times \dot{\boldsymbol{q}}+\boldsymbol{\omega} \times \boldsymbol{\omega} \times \boldsymbol{q}=\frac{\partial \mathcal{U}}{\partial \boldsymbol{q}}$
where $\boldsymbol{q}$ denotes the satellite position in the rotating, bodyfixed frame and all time derivatives are taken with respect to this rotating frame (leading to the inclusion of Coriolis and centripetal accelerations). There are two general cases that occur for these systems. One is that the small body is in an arbitrary rotation state, tumbling in inertial space and following the torque-free solution. Then the angular velocity vector is time-periodic in the body-frame and the term $\dot{\omega} \neq 0$. Dynamics in such a case have been explicitly studied for the tumbling asteroid Toutatis [6], and some representative orbits are shown in Figs. 4 and 5.

The more usual case is for the small body to be uniformly rotating about its maximum moment of inertia. Then the angular velocity vector is a constant, $\dot{\omega}=0$, and the equations of motion in the body-frame become timeinvariant. In this case the dynamical system has a Jacobi integral which is conserved, expressed as
$J=\frac{1}{2} v^{2}+\frac{1}{2}(\boldsymbol{\omega} \times \boldsymbol{r}) \cdot(\boldsymbol{\omega} \times \boldsymbol{r})-\mathcal{U}$
There are further implications for the time-invariance of these equations of motion beyond the existence of a Jacobi integral. First, the existence of this integral allows the use of zero-velocity surfaces to be defined and used to establish stability of motion. Second, this implies that equilibrium points can exist and that periodic orbits are dense in the phase space of this problem and can be used to study the phase space structure of this system.

### 4.2. Equilibria and periodic orbits

The computation and study of special solutions to the equations of motion can form a strong basis for understanding the stability of motion in the small body orbiter problem. When a periodic orbit or equilibrium point is found, there are well defined methods for determining the stability of motion in the vicinity of these solutions. The stability of these solutions generically applies to the neighborhood of that point in phase space as well, and thus informs whether motion will remain in the vicinity of an orbit for some time period or whether it will rapidly depart on an unstable trajectory.

For the more general case when the asteroid is tumbling, periodic orbits can only have periods commensurate with the time-periodic angular velocity period in the body-fixed frame. Due to this equilibria are not possible, and periodic orbits in general are "isolated" in phase space, only existing when specific resonances between the orbit period (in the body-fixed frame) and the angular velocity period exist. Fig. 5 shows some representative periodic orbits for such a case.

When the body is uniformly rotating it is possible to define equilibrium points and families of periodic orbits. The asteroid Eros, for example, has been extensively analyzed in terms of the periodic orbit structure about that body $[28,8]$. Indeed, the orbits shown in Fig. 1 can in part be explained with this analysis. The stable orbit presented actually is in the vicinity of the "closest, stable direct periodic orbit" about that body. Thus, by starting an orbit in its vicinity (where the usual circular speed happens to be close to the true periodic orbit speed at that orientation), one can have some confidence that the


Fig. 4. A stable orbit about asteroid Toutatis, shown in the body-fixed frame. The orbit plane is dragged with the tumbling motion of the asteroid.


Fig. 5. Periodic orbits about asteroid Toutatis, shown in a body-fixed frame. The top orbits have period equal to the Toutatis angular velocity vector in the body frame. The bottom orbit has period equal to twice this.
resulting motion may stay in the vicinity of that periodic orbit. For this case, motion close to a strongly distended gravitational field, the radius and velocity of an orbiter is found to vary significantly as a function of its phase angle with respect to the mass distribution. Thus, by shifting the phase angle by $45^{\circ}$ but keeping the speed constant, the initial conditions are moved away from the relatively narrow stability zone about the stable periodic orbit family members at that distance from the asteroid. Thus, the resultant motion is highly unstable and the orbiter escapes within a few orbits. Fig. 6 shows a direct application of periodic orbit stability computations to mapping out the phase space about a uniformly rotating body. By computing families of periodic orbits that transitioned from planar to out-of-plane and tracking the stability of these orbits, the inclination at which these orbits transitioned from unstable to stable enables a heuristic limit on orbit semi-major axis and inclination for stability to be developed.

The computation of equilibrium points (i.e., $1: 1$ resonant orbits) also allows for such analyses to be made. It is significant to note, however, that the majority of asteroids have only unstable equilibria in their vicinity [15]. These relative equilibria generally lie near the body's equatorial


Fig. 6. Stability regions as a function of orbit semi-major axis and inclination about Eros [8].


Fig. 7. Pole-down view of asteroid Betulia and its 6 equilibrium points [9].
plane (the plane perpendicular to the body's axis of maximum moment of inertia), with the number of relative equilibria being controlled by the shape of the body. Most small bodies have just four relative equilibria, however cases have been analyzed that have several additional equilibria. As an example of this, Fig. 7 shows a pole-down view of the asteroid Betulia, which has a triangular shape when viewed from this direction [9]. This body has six relative equilibrium points, all near its equatorial plane.

For a given shape and spin rate, the distance of these points relative to the asteroid is a function of the body density (which often must be assumed). For larger densities the equilibrium points move further from the body and the possibility for at most half of the equilibrium points to become stable occurs. As the density decreases these points all move toward the central body and in general all become unstable.

Generally speaking, the placement of a satellite in or near any of these relative equilibria is not a feasible mission design. First, most of these points are unstable for small bodies of interest. Further, the timescale of their
instability tends to be a factor of a few faster than the asteroid's rotation period. Thus at Eros, for example, which has a 5.27 h period, the characteristic instability time of its four equilibrium points ranges from 40 to 100 min [28]. Such a rapid divergence of a satellite from its nominal location would be extremely challenging to control from the ground. Similar results are found for almost all small bodies studied to date.

### 4.3. Zero-velocity surfaces

For uniformly rotating bodies the existence of the Jacobi integral allows for zero-velocity surfaces to be defined and used to design trajectories that cannot impact with the central body, of significant interest for closeproximity orbit designs. If the $z$-axis is aligned with the angular velocity vector, then the zero-velocity surfaces can be stated in an implicit form as
$\frac{1}{2} \omega^{2}\left(x^{2}+y^{2}\right)+\mathcal{U}(x, y, z) \geq-C$
where $C=J(x, y, z, \dot{x}, \dot{y}, \dot{z})$ is the value of the Jacobi constant. The Jacobi integral can also be stated in terms of a Tisserand-condition like form in terms of the osculating orbit elements of periapsis radius, eccentricity and inclination
$\frac{\mu(1+e)}{2 r_{p}}-\omega \sqrt{\mu r_{p}(1+e)} \cos (i)-\mathcal{U}\left(r_{p}, \hat{\boldsymbol{r}}\right)=J$
where the gravitational force potential is evaluated at a position vector interpreted as periapsis, and $r_{p}=a(1-e)$. This allows the osculating orbit elements to be directly related to the Jacobi constant.

Similar to the restricted 3-body problem, the applicability of the zero-velocity surfaces as sharp constraints for motion degrades as the inclination increases, thus this is most applicable for direct motion (i.e., for inclinations close to $0^{\circ}$ ). Using these surfaces it is possible to delimit relative locations and speeds that ensure that a spacecraft will not be able to impact with the asteroid surface. Fig. 8 shows such an application for the asteroid Eros, delimiting the combination of periapsis radius and eccentricity to ensure that impact with the asteroid cannot occur. Note the consistency with Fig. 1, where impact of any of these orbits cannot be ruled out a priori.

### 4.4. Analytical constraints

Finally, it is possible to place sharp analytical limits and predictions for long-term motion in the gravity dominant regime through the application of traditional celestial mechanics techniques of averaging, and other related approaches. Using these techniques at small bodies presents a challenge for these approaches as the perturbation gravity coefficients are generally much larger than have been practically dealt with for precision analytical theories of motion. Thus, inclusion of higher order terms may still not yield a fully convergent solution, taking away some of the motivation for the development of such higher-order theories. Instead, relying on firstorder estimates of these results can often yield results of


Fig. 8. Stability limits to ensure non-impact of a satellite about Eros [28].
sufficient precision to enable the first round of mission design, with iterations relying on the use of detailed simulations. Recall the simple formula for the precession of an orbiter's longitude of the ascending node and argument of periapsis due to the oblateness gravity coefficient $\mathrm{C}_{20}$.
$\dot{\Omega}=\frac{3 n C_{20} R^{2}}{2 p^{2}} \cos i$
$\dot{\omega}=\frac{3 n C_{20} R^{2}}{2 p^{2}}\left(\frac{5}{2} \sin ^{2} i-2\right)$
As a simple way to compare the orbital environment about a small body with an Earth orbiter, compare the magnitude of ( $3 n\left|C_{20}\right| R^{2}$ )/2a $a^{2}$ of the NEAR spacecraft about Eros with those of an Earth orbiter. Making this comparison at a similar distance from each body (as measured in mean radii) shows that the precession rate about Eros is over 200 times faster than an Earth orbiter's precession rate.

A more serious issue for analytical theories for orbiters is the strong influence that the $C_{22}$ gravity coefficient has on the orbital dynamics about a strongly-distended small body. For Earth orbiters, this is a very small perturbation and its effect generally averages out over time (except for orbits in a $1: 1$ resonance). At small bodies this gravity coefficient is extremely important and is what causes most of the observed chaotic motion in these systems. In particular, in Fig. 1 both the impacting and the escaping trajectories occur so rapidly because of the interaction of the satellite with Eros' $C_{22}$ gravity coefficient. The challenge is that averaging procedures no longer work for such time-varying components of a gravity field, as the resultant change in an orbit depends sensitively on initial conditions. Taking a different analytic approach to this
problem, it is possible to develop explicit predictions on the expected change in an orbit's state over one interaction with such a rotating gravity field [19]. Ignoring terms that can be demonstrated to be small, it is possible to predict the change in an object's energy and angular momentum over one orbit about a system as
$\Delta E=-6 \omega C_{22} R^{2} \sqrt{\frac{\mu}{p^{3}}} \cos ^{4}(i / 2) \sin 2(\theta) I_{2}^{1}\left(r_{p}, e\right)$
$\Delta G=-6 C_{22} R^{2} \sqrt{\frac{\mu}{p^{3}}} \cos ^{4}(i / 2) \sin 2(\theta) I_{2}^{1}\left(r_{p}, e\right)$
$I_{2}^{1}=2 \int_{0}^{\pi}(1+e \cos f) \cos (2 f-2 \omega t) d f$
where $E$ is the Keplerian energy, $G$ is the angular momentum magnitude, $\omega$ is the asteroid rotation rate, $C_{22} R^{2}$ is the gravity coefficient times the normalizing radius squared, $\mu$ is the asteroid gravitational parameter, $p$ is the orbit parameter, $i$ is the inclination from the equatorial plane, $\theta$ is the longitude that the periapsis makes in the body-fixed frame, measured from the long-end of the asteroid. The integral $I_{2}^{1}$ is a Hanson Coefficient [20], in the integral $t$ is time and $f$ is true anomaly. The integral can be expressed as a function of periapsis radius $r_{p}$ and orbit eccentricity $e$, and is shown in Fig. 9.

From these functional relations it can be noted that orbit-to-orbit change in a trajectory depends on where in the body-fixed frame periapsis passage occurs. When it lies in the first and third quadrants (i.e., over the leading edges of the rotating body) the energy and angular momentum are both decreased, while when over the trailing edges (the second and fourth quadrants) the energy and angular momentum are boosted. These changes are strong enough to change an orbit's energy from negative (bound) to positive (unbound) over one passage (cf., Fig. 1).

These results also suggest a mission design strategy. As the inclination increases (up to retrograde, or $180^{\circ}$ in the limit) these effects become diminished. There are other


Fig. 9. Integral $I_{2}^{1}$ as a function of normalized radius of periapsis and eccentricity. Note that $\left|I_{2}^{1}\right| \leq \pi$ [19].
terms that contribute to changes in the energy and angular momentum as the orbit becomes retrograde, however they tend to have a much smaller magnitude. Thus, an effective strategy for minimizing the perturbations from these terms is to place spacecraft in retrograde orbits about the body, which is precisely the strategy taken by the NEAR mission at Eros [17]. This analysis also provides an analytical understanding for the stability of high inclination orbits apparent in Fig. 6.

## 5. Solar dominated regime

Now consider orbital mechanics about small bodies when the Sun is the dominant source of perturbations. There are two primary effects from the Sun that influence motion, its gravitational attraction and the solar radiation pressure (SRP) that acts on an orbiter. Except for objects with very low area to mass ratios, SRP is usually dominant over tides for spacecraft orbit dynamics. There are two items of interest when dealing with solar perturbations. First is under what conditions an orbiter will be able to stay bound to a small body in the presence of these additional forces. Second is how their orbital dynamics will evolve over time due to these perturbations. The methodology for answering these questions are quite different, and each is reviewed below. The assumption for this section is that the central body is spherical and modeled as a point mass. In later sections this assumption is relaxed.

### 5.1. Escape limits

Two different approaches have been taken in the literature to establish limits for orbital motion about a spherical body in orbit about the Sun and subject to solar radiation pressure (SRP). Using a non-rotating model that does not include the solar gravitational attraction, Dankowicz [5] established a conservative maximum limit for when a satellite would escape from its orbit about a central body. This was expressed in terms of orbit semimajor axis by Scheeres [22] and gives an upper limit beyond which a spacecraft will escape
$a_{\text {Max }}=\frac{\sqrt{3}}{4} \sqrt{\frac{\mu}{\beta}} d$
Using a different approach, with a more realistic model incorporating the elliptic motion of the small body about the Sun and the solar gravitational attraction, Scheeres and Marzari [25] derived an exact necessary condition for escape from a spherical body in orbit about the Sun. The full criterion is complex, but it can be simplified under a few assumptions to yield a similar form to the Danckowicz bound. It is a sufficient condition for stability, and thus for a spacecraft to be definitely bound in orbit about a small body the semi-major axis should be less than
$a<\frac{1}{4} \sqrt{\frac{\mu}{\beta}} d$
and differs from the other result by a factor of $\sqrt{3}$. The true limit depends on a number of additional parameters, and this sufficiency condition has been validated with
numerical simulations and shown to be sharp for some orbit geometries [25].

Note that these limits explicitly predict that as a small body moves on an elliptical orbit about the Sun that it is possible for a previously stable orbiter to escape from the body, as $d$ will decrease as perihelion is approached. In Fig. 2 this is explicitly shown, as the orbit is initially bound to the body but abruptly escapes once the small body's distance from the Sun passes a given limit. Similarly, in Fig. 3 the initial orbit semi-major axis is chosen to just barely violate the appropriate stability limit, leading to immediate escape for the larger orbit and to a bound orbit for the closer one (which eventually impacts). These bounds serve as a crucial design tool for developing mission plans about smaller bodies.

### 5.2. Secular orbital evolution

Using the above limits to ensure bounded orbits about the central body, it becomes possible to perform an averaging analysis to extract the averaged equations of motion for the orbit constants of a satellite. These can then be solved to develop specific predictions on the secular evolution of orbits subject to SRP. The first-order average of the SRP potential yields a particularly simple result
$\overline{\mathcal{R}}_{S R P}=\frac{1}{2 \pi} \int_{0}^{2 \pi} \mathcal{R}_{S R P} d M$
$\overline{\mathcal{R}}_{S R P}=-\frac{3}{2} \frac{\beta}{d^{2}} \hat{a} \hat{\boldsymbol{d}} \cdot \boldsymbol{e}$
where the average is over one orbit of the satellite about the small body and $\boldsymbol{e}$ is the eccentricity vector. This problem has been studied in the past, with Hénon and Mignard [11] first noting that the averaged equations can be solved in closed form for a non-rotating SRP force (i.e., a body not moving relative to the Sun), and Scheeres [18] explored this solution and generalized it to elliptic motion about the Sun. Richter and Keller [13] developed a simpler solution technique for the non-rotating case and Scheeres [22] again generalized this case to elliptic motion about the Sun, and explored several aspects of its solution that are relevant for mission design, reviewed here.

The solution procedure adopted by Richter and Keller and further generalized by Scheeres uses the eccentricity vector and the scaled angular momentum vector as orbital elements. Since the semi-major axis is conserved for the secular potential defined above, the usual angular momentum vector is scaled by $\sqrt{\mu a}$, or $\boldsymbol{r} \times \boldsymbol{V} / \sqrt{\mu a}=\boldsymbol{h}$. Then the magnitude of $\boldsymbol{h}$ is $\sqrt{1-e^{2}}$ and the scaled angular momentum and eccentricity vectors satisfy the identities $\boldsymbol{e} \cdot \boldsymbol{h}=0$ and $\boldsymbol{e} \cdot \boldsymbol{e}+\boldsymbol{h} \cdot \boldsymbol{h}=1$. With these definitions, the eccentricity and scaled angular momentum vector of a small body orbiter subject to solar radiation pressure varying with the elliptic orbit of the small body about the Sun can be solved in closed form, with the details given in Scheeres [22]. When transformed into a rotating frame with the small body's heliocentric true anomaly as the independent variable the equations are reduced to a time-invariant linear system, and thus can be solved in
closed form. This solution is expressed as

$$
\left[\begin{array}{l}
\boldsymbol{e}  \tag{18}\\
\boldsymbol{h}
\end{array}\right]=\Phi(\psi)\left[\begin{array}{l}
\boldsymbol{e}_{o} \\
\boldsymbol{h}_{o}
\end{array}\right]
$$

where the " 0 " subscript denotes an initial condition. The matrix $\Phi$ is an orthonormal $6 \times 6$ matrix and has components

$$
\begin{align*}
\Phi(\psi)= & \cos (\psi) I_{6 \times 6}+(1-\cos (\psi)) \\
& \times\left[\begin{array}{cc}
\cos ^{2} \Lambda \hat{\boldsymbol{z}} \hat{\boldsymbol{z}}+\sin ^{2} \Lambda \hat{\boldsymbol{d}} \hat{\boldsymbol{d}} & -\sin \Lambda \cos \Lambda(\hat{\boldsymbol{z}} \hat{\boldsymbol{d}}+\hat{\boldsymbol{d}} \hat{\boldsymbol{z}}) \\
-\sin \Lambda \cos \Lambda(\hat{\boldsymbol{z}} \hat{\boldsymbol{d}}+\hat{\boldsymbol{d}} \hat{\boldsymbol{z}}) & \cos ^{2} \Lambda \hat{\boldsymbol{z}} \hat{\boldsymbol{z}}+\sin ^{2} \Lambda \hat{\boldsymbol{d}} \hat{\boldsymbol{d}}
\end{array}\right] \\
& +\sin (\psi)\left[\begin{array}{cc}
-\cos \Lambda \tilde{\tilde{\boldsymbol{z}}} & \sin \Lambda \tilde{\boldsymbol{d}} \\
\sin \Lambda & -\cos \Lambda \tilde{\boldsymbol{z}}
\end{array}\right] \tag{19}
\end{align*}
$$

where $\psi=f / \cos \Lambda, \hat{\boldsymbol{z}}$ is the axis about which the asteroid revolves about the Sun (perpendicular to $\hat{\boldsymbol{d}}$ ), two multiplied vectors is a dyad, and $\tilde{\tilde{z}}$ signifies the skew-symmetric cross-product tensor. The parameter $\Lambda$ is defined as a function of the asteroid mass, orbit satellite's orbit and SRP parameter as
$\tan \Lambda=\frac{3 \beta}{2} \sqrt{\frac{a}{\mu \mu_{\text {sun }} a_{s}\left(1-e_{s}^{2}\right)}}$
where $\mu_{\text {sun }}$ is the Sun's gravitational parameter and $a_{s}$ and $e_{s}$ are the asteroid's heliocentric semi-major axis and eccentricity. The parameter $\Lambda$ is a constant and is well defined for any asteroid and satellite in orbit about it. As the SRP perturbation becomes large $\Lambda \rightarrow \pi / 2$, while $\Lambda \rightarrow 0$ for a weak SRP perturbation. The NEAR spacecraft at Eros had a small value for this parameter, while the Hayabusa spacecraft at Itokawa and the Rosetta spacecraft at comet 67P/CG will have large values greater than $45^{\circ}$. Note that the solution is periodic in $\psi$, and that over one asteroid year the SRP solution advances $2 \pi / \cos \Lambda$ times. Thus for a strongly perturbed system this solution will repeat frequently, and for a weakly perturbed system will repeat approximately once per year. Note that the solution is expressed relative to a frame rotating with the Sun about the small body and incorporates the effect of varying SRP strength with distance.

Despite its simple form, the solutions for eccentricity, inclination, longitude of the ascending node and argument of periapsis are quite complex and change drastically as a function of their initial conditions and parameter $\Lambda$. Two general cases are discussed. First, if the angular momentum vector of the satellite is parallel to the vector $\hat{\boldsymbol{z}}$ (i.e., the orbit is in the ecliptic) and the orbit is initially circular the evolution of eccentricity will follow the equation
$e(\psi)=2 \sin \Lambda|\sin (\psi / 2)| \sqrt{1-\sin ^{2} \Lambda \sin ^{2}(\psi / 2)}$
The evolution of the eccentricity as a function of $\psi$ is shown in Fig. 10 for a range of $\Lambda$. Note the complex behavior and that the maximum value of eccentricity goes to unity when $\Lambda \geq 45^{\circ}$. This result precisely explains the bound orbit that impacts shown in Fig. 3. Even though an orbit's eccentricity goes through unity (i.e., its periapsis radius goes to zero) does not mean that it will immediately impact, as periapsis may increase above the asteroid surface by the time the satellite passes through periapsis again. However, if the


Fig. 10. Time histories of eccentricity for a range of perturbation strengths [22].


Fig. 11. Frozen terminator orbit propagated in a numerical simulation over a full asteroid year [22].
eccentricity repeatedly goes through unity (as in the example) it becomes likely that it will eventually impact.

The most important aspect of this solution, however, is that it admits constant orbital elements for special initial conditions. Specifically, if the angular momentum is initially directed toward or away from the Sun, the eccentricity vector directed above or below the ecliptic plane (respectively), and the eccentricity chosen to equal $\cos \Lambda$, then the orbit remains constant on average. As described, these orbits lie in the terminator plane of the orbit, although they are slightly displaced away from the Sun [16]. Thus, these orbits automatically track the Sun, due to SRP torques acting on the orbit angular momentum. Also significant, the frozen eccentricity becomes more circular as the orbit perturbation becomes stronger, and thus these orbits are well defined for
very small bodies, so long as the orbit remains bounded. Finally, the orbit remains frozen even as the asteroid travels through perihelion and aphelion, due to the balance between true anomaly rate of change and variation in SRP, as both vary as $1 / d^{2}$. Fig. 11 shows a frozen terminator orbit about asteroid 1989ML numerically simulated over a full asteroid year, incorporating the full elliptic motion of that body about the Sun.

These orbits serve as the nominal choice for any orbital mission to a small asteroid, as almost all other orbits about these bodies will suffer large variations in eccentricity and the other orbit elements. In a recent paper, Shupe and Scheeres [29] probe the minimum asteroid size for when such orbits remain feasible. For an Orion-class spacecraft they were able to find a feasible range of orbits about a body as small as 10 m across.

## 6. Mixed results

The above analyses are each idealized in that they neglect the effect of the other perturbation. For real systems, however, both gravity and SRP perturbations are present and can provide real limitations on the mission design results discussed above. Analyzing both gravity and SRP perturbations jointly is difficult, and only limited analytical results have been found [22]. In one set of analyses it was shown that despite the joint effects of gravity and SRP it would have been feasible for the Japanese Hayabusa spacecraft to orbit about the asteroid Itokawa [24]. Thus, the mission design principles outlined here can still be applied and used for the initial design of a close proximity orbital mission at a small body. As a case in point, in the following such an analysis is performed for the Rosetta spacecraft at Comet 67P/Churyumov-Gerasimenko, using previously published shape models of that body.

## 7. Rosetta at 67P/CG

### 7.1. Model for Rosetta and 67P/CG

Table 1 summarizes the various gravitational and nongravitational parameters used to describe the Rosetta spacecraft at comet $67 \mathrm{P} /$ Churyumov-Gerasimenko (67P/CG). The derivation of these results is outlined in the following. All of the following simulations incorporate all of the perturbations arising from the above models, while the specific design results only focus on the ideal equations.

The model for the Rosetta spacecraft is not precise but based on descriptions of the satellite size and probable mass taken from the gray literature. The spacecraft is rather massive with its total mass ranging from $2300 \rightarrow 1700 \mathrm{~kg}$ over the life of the mission, however the total projected area that the satellite will present to the Sun is also large, at approximately $77 \mathrm{~m}^{2}$. Thus this yields an area to mass ratio that ranges from 0.033 to $0.045 \mathrm{~m}^{2} / \mathrm{kg}$. For definiteness in the following a value of 0.0385 is taken with a zero reflectance model. This leads to a value of $\beta=3.85 \times 10^{6} \mathrm{~km}^{3} / \mathrm{s}^{2}$ or $1.711 \times 10^{-10} \mathrm{~km} \mathrm{AU}^{2} / \mathrm{s}^{2}$. More useful sometimes is a direct comparison of $\beta$ to the solar gravitational parameter of $\mu_{\text {Sun }} \sim 1.33 \times 10^{11}$, yielding $\beta / \mu_{\text {Sun }}=2.89 \times 10^{-5}$.

Table 1
Assumed Rosetta and 67P/CG parameters.

| Parameter | Value | Units |
| :--- | :--- | :--- |
| $\beta$ | $3.85 \times 10^{6}$ | $\mathrm{~km}^{3} / \mathrm{s}^{2}$ |
| tan $\Lambda$ | $1.08 \sqrt{a}$ | $(-)$ |
| Mean radius | 1.9 | km |
| Density | 0.37 | $\mathrm{~g} / \mathrm{cm}^{3}$ |
| $\mu$ | $7 \times 10^{-7}$ | $\mathrm{~km}^{3} / \mathrm{s}^{2}$ |
| $C_{20}$ | -0.4599 | $\mathrm{~km}^{2}$ |
| $C_{22}$ | 0.0876 | $\mathrm{~km}^{2}$ |
| Period | 12.55 | h |
| Obliquity | 138 | $\circ$ |
| Semi-major axis | 3.468 | AU |
| Eccentricity | 0.64 | $(-)$ |
| Perihelion | 1.25 | AU |
| Aphelion | 5.69 | AU |



Fig. 12. Shape model of Comet 67/P CG [7].
The comet model is based on observations of this target and, while of low overall resolution, should be a reasonable stand-in for the real shape pending high precision observations of the comet by the Rosetta spacecraft. The specific shape and other information is taken from Lamy et al. [7]. The shape has a mean radius of 1.9 km and an assumed density of $0.37 \mathrm{~g} / \mathrm{cm}^{3}$. Taken together they give a total gravitational parameter of $\sim 7 \times 10^{-7} \mathrm{~km}^{3} / \mathrm{s}^{2}$. Its spin period was estimated to be 12.55 h and its spin state assumed to be uniform rotation, with its pole at a $138^{\circ}$ obliquity. Fig. 12 shows this shape model.

From the shape the constant density gravity coefficients $C_{20}$ and $C_{22}$ can be computed, shown in Table 1.

Finally, the comet orbit is specified by a semi-major axis of 3.468 AU and an eccentricity of 0.64 . Thus, over a course of its year its distance from the Sun ranges from 1.25 to 5.69 AU, a sizable range. Given these specifications it is possible to define a relationship for the angle $\Lambda$ as
$\tan \Lambda=1.08 \sqrt{a}$
$\tan \Lambda=1.488 \sqrt{a / R}$
where $a$ is specified in kilometers and $R$ is the mean radius in kilometers. Thus, at the mean radius of the comet the angle $\Lambda \sim 56^{\circ}$ and grows larger as the semimajor axis increases. Recall from Eq. (21) that for $\Lambda \geq 45^{\circ}$ all planar solutions will go through a unity value of eccentricity, barring other perturbations. Thus for all orbit radii about comet $67 \mathrm{P} / \mathrm{CG}$ the solar radiation pressure will be an important and potentially dominant force.

### 7.2. Analytic results

First consider the limiting semi-major axis for the spacecraft to be in a bound orbit about the body. Using the previously defined values $a_{\max }=27.7 d \mathrm{~km}$, where $d$ is the comet-Sun distance in AU. Thus this limit varies from 34.6 to 157.6 km between perihelion and aphelion, respectively.

Now consider motion close to the body. The comet has four relative equilibria about it, shown in Fig. 13. All four of these points are unstable, and thus are not viable candidates for placement of an orbiter or any other sort of vehicle. Also shown in this figure are the projection of the zero-velocity surfaces onto the equatorial plane of the comet. The colors indicate the surface slopes on the comet, and range from $30^{\circ}$ (red) to near $0^{\circ}$ (blue). Note that these slopes are relatively coarse, given the poor resolution of the shape model. The histogram of slopes over the current shape model (incorporating both gravitational attraction and centripetal accelerations [27]) is shown in Fig. 14.

Eq. (11) can be used to estimate the effect of the $C_{22}$ gravity field on close proximity dynamics. Assuming a conservative value of $I_{2}^{1}=\pi$ (which is close to the expected value for an orbit close to the surface of the comet) an orbit with semi-major axis equal to the comet's mean radius can suffer a change in its energy (and of its semi-major axis) on the order of $20 \%$ from orbit to orbit. This is an extremely large perturbation and indicates that direct orbits close to the comet nucleus will suffer large


Fig. 13. Zero-velocity curves and equilibrium points about comet 67P/CG. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 14. Slope histogram for comet 67P/CG.
perturbations. Note that polar orbits will have a quarter of this perturbation and that the strength of these perturbations (in terms of relative change in orbit energy) will scale as $1 / \sqrt{a}$. Relevant to this, it is found in Scheeres et al. [28] that a reasonable limit in fluctuations in the orbit energy for stability is that they be less than $\sim 5 \%$. For a terminator orbit, the inclination will be on the order of $42^{\circ}$ or greater with a scaling of the fluctuation term on the order of 0.75 . Thus, to have the orbit-per-orbit fluctuations to be reduced to $5 \%$ would require an orbit semi-major axis on the order of 17 km . This is not a rigorous limit, however, and only provides some guidance on where the approximate minimum orbit should be.

Finally, the highly oblate shape of the comet will also affect the secular rates of the longitude of the node and the argument of periapsis when close to the body. Using the classical formula for the longitude of the node to estimate the order of magnitude of this effect, allows its coefficient $3 n / 2 /\left[a^{2}\left(1-e^{2}\right)\right]\left|C_{20}\right|$ to be used as a measure. For an orbiter at the surface of the comet this angular rate is $2.2 \times 10^{-4} \mathrm{rad} / \mathrm{s}$, or $45^{\circ} / \mathrm{h}$. This rate will decay as $1 / a^{3.5}$.

### 7.3. Direct simulations

In the following a range of orbits about the comet nucleus are simulated, using the above perturbations. This allows the mission design results and limits outlined in this paper to be verified, and to arrive a set of feasible trajectories that an orbiter could take at this comet. Fig. 15 shows the range of distances between the comet and the Sun over the simulation timescale. Note that perihelion occurs at approximately 7500 h (312.5 days) into the simulation.

### 7.3.1. Direct orbits

Orbits in close proximity to the comet are probed first. It is expected that direct orbits close to the nucleus should be highly unstable, which is verified with numerical integrations. As the initial orbit radius becomes higher, these orbits are stable over longer periods of time, however the effect of the SRP perturbations also influences their motion, and if left uncontrolled drives these orbits into instability. Figs. 16


Fig. 15. Comet $67 \mathrm{P} / \mathrm{CG}$ radial history over the simulations presented below.


Fig. 16. Semi-major axes of a number of direct orbits about 67P/CG.
and 17 show the semi-major axis and eccentricity of a number of initially circular orbits propagated up to 100 days. Note the instability of the closest orbit, but that higher orbits may be stable over longer time spans. All of the orbits experience a growth in eccentricity, however. Note that these orbits are in the comet nucleus' equatorial plane, which is inclined by up to $48^{\circ}$ from the comet's heliocentric orbit plane. From these figures note that larger direct orbits can be maintained, although if uncontrolled will have unacceptably high eccentricities.

### 7.3.2. Retrograde orbits

As a possible remedy to the instability of direct orbits, consider the dynamics of retrograde orbits about the comet. Very close orbits are stable, as shown in Figs. $18-20$. As the comet may be outgassing, these orbits may be, in fact, too close for comfort to the surface. Also, even though they are in close proximity, they are not ideal for deployment of a lander as the relative speed with respect to the comet surface is relatively large as the spacecraft and comet nucleus are counter-rotating. Note that when in close proximity to the gravity field, the large


Fig. 17. Eccentricities of a number of direct orbits about 67P/CG.


Fig. 18. Rosetta retrograde orbits: semi-major axis.


Fig. 19. Rosetta retrograde orbits: eccentricity.
$C_{20}$ coefficient causes the argument of periapsis and longitude of the node to have very fast precession rates. This allows these close orbits to forestall the secular growth in eccentricity that would otherwise occur due to the SRP perturbation. As larger semi-major axis values are considered, however, an increase in the secular


Fig. 20. Rosetta retrograde orbits: trajectory plots.
growth of the eccentricity does occur and is clearly seen in Fig. 19. These lead to a decrease in the periapsis and close interactions with the rotating mass distribution. While these are not as strong as the direct interactions, they can affect the orbit dynamics, which is seen in the fluctuations of the semi-major axis in Fig. 18. Note the clear correlation between the fluctuations in eccentricity and proximity to perihelion (comparing times with Fig. 15). Also note that orbits at a semi-major axis of 5 km remain reasonable over a long time span, and only suffer larger excursions in eccentricity when around perihelion. Even at a distance of 7.5 km , however, the fluctuations in eccentricity already become significant well before perihelion and would require some active monitoring and control. Thus, retrograde orbits could be considered for operational use at the comet, pending investigation of the effects of outgassing.

### 7.3.3. Terminator orbits

Finally, consider the dynamics of terminator orbits for the Rosetta spacecraft about the comet. It has been noted that these orbits will remain "frozen" in the frame rotating with the comet Sun-line. Even though the Sunline has a wide range of rotation rates, the balance between the SRP torque and the true anomaly rate is such that the orbiter will nominally remain close to the terminator plane throughout the comet year, if designed correctly. Note that the SRP angle $\Lambda$ ranges from $56^{\circ}$ at the surface to $81^{\circ}$ at 35 km , the limit for bounded motion at perihelion. Thus the eccentricity of these orbits should range from 0.56 to 0.155 at the upper limit of semi-major axis. For orbits away from perihelion, this upper limit can be higher and the eccentricity even lower.

In addition to this upper limit on orbit size, arising from the limits for bounded motion about the nucleus, there will also be a lower limit in orbit size related to the interaction of the spacecraft with the comet gravity field.

Recall that the secular orbit rates in longitude and argument of periapsis are quite large. Thus, for a low enough semi-major axis, the secular rate of the orbit node due to the $C_{20}$ and higher order zonals can start to act against the torque on the orbit that SRP provides. This mismatch can cause the orbit to librate about the terminator plane, allowing the SRP perturbation to start acting on the orbit eccentricity. This in turn causes the orbit periapsis to drop closer to the mass distribution and can excite fluctuations in orbit energy and angular momentum due to interactions with the $C_{22}$ coefficient. Thus, as the orbit radius drops it is expected that there will be a lower limit for long-term orbit stability. This has been analyzed in some detail in Scheeres [22]. Figs. 21-23 show numerical integrations of a series of Rosetta orbits started at different frozen orbits about comet 67P/CG. The initial conditions were all chosen in accordance with $e=\cos \Lambda$ for the given semi-major axes, and thus would have had fixed values of eccentricity for orbits about a sphere.

Looking at Fig. 23 explicitly, note that the orbit at a distance of $\sim 35 \mathrm{~km}$ is bounded, and shows mild signs of instability at perihelion. The larger orbit is definitely stripped out of orbit by the time perihelion is reached. Note that with such a high comet eccentricity, the rate of decrease in the orbit distance $d$ is very high around perihelion, and thus orbits that would eventually be stripped out of orbit may not have sufficient time to realize this final state. At the lower range of semi-major axis a different behavior is seen, with interactions between the mass distribution and the orbiter eventually destabilizing their orbits and leading to impact. The interactions in this regime are difficult to analyze-as a case in point the intermediate orbit actually impacts prior than the closer orbit. The specific reason for this may be complex and relate to the detailed dynamical evolution of each case.


Fig. 21. Rosetta terminator orbits: Sun-view.


Fig. 22. Rosetta terminator orbits: transverse view.


Fig. 23. Rosetta terminator orbits: semi-major axis.

One conclusion which can be drawn for this current model is that orbits with semi-major axis between 15 and 35 km are definitely bound to the nucleus and, barring other perturbations such as comet outgassing, would remain in orbit about the nucleus indefinitely. Given this robust range of orbits, it is easy to assert that the terminator class of orbit are a feasible orbit design with respect to long-term stability.

### 7.4. Outgassing

Finally, a few observations about outgassing at comets are given. There have been a few analyses of orbital motion about comets accounting for outgassing effects in addition to solar radiation pressure and nucleus gravity effects. Scheeres et al. [26] study the orbital dynamics for the Rosetta orbiter about the original target, Wirtanen. Papers by Byram et al. [4,3] have studied aspects of comet outgassing models, navigation and mission design
questions for cometary orbiters. More recently Mysen et al. [12] have started modeling more complex outgassing fields and, in preparation for the rendezvous of Rosetta with comet 67P/CG, have begun to develop a high fidelity model.

The truth of the matter is that the outgassing environment close to a comet nucleus is not well understood. Thus, the data which the Rosetta spacecraft acquires on these effects will be of fundamental importance in understanding how spacecraft interaction with these fields should be modeled. Despite this large uncertainty there are still a few principles which can indicate how stable orbits could be developed. The fundamental observation is that the cometary activity should be most active at the sub-solar point and later in the day, while being minimum at the Sun-rise terminator. Of course, this depends on the orientation of the comet rotation state as well. Depending on the thermal inertia of the nucleus, the outgassing may also be past its peak at the Sun-set terminator. This sets up the terminator orbits as having an additional advantage, as they may be exposed to the least extreme outgassing environment, a similar conclusion to that reached in Scheeres et al. [26] and studied further in Byram et al. [3]. We do not comment on the outgassing environment beyond this.

## 8. Conclusion

The orbital environment about small solar system bodies such as asteroids and comets has been analyzed in detail. Specific mission design solutions have been formulated across the range of body size and shape. As a specific application, the orbital dynamics environment which the Rosetta spacecraft will encounter at Comet 67P/Churyu-mov-Gerasimenko was studied and specific predictions and design suggestions made. Orbital mechanics about these bodies serve as a challenging problem for astrodynamics, and present real opportunities for the continued advancement of the field in pursuit of better understanding motion in these extreme environments.

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    E-mail address: scheeres@colorado.edu

